Spin and Charge Qubits in Si/SiGe Quantum dots

by

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Abstract

In this thesis, we study the spin and charge properties of Si/SiGe double dot system and demonstrate coherent quantum control of few-electron qubits.

First, we use a pulsed magneto-spectroscopy method to measure the excited-state spectrum of a Si/SiGe quantum dot. The application of this technique in a two-electron quantum dot allows us to identify the spin of the lowest three eigenstates, which are $S$, $T_-$ and $T_0$, in different field values. We extract the singlet-triplet energy splitting, an essential parameter for spin qubits, from the magneto-spectroscopy data and find it tunable by lateral displacement of the dot, which is realized by changing two gate voltages on opposite sides of the device. We present calculations showing the data are consistent with a spectrum in which the first excited state of the dot is a valley-orbit state.

Next, making use of the two-electron singlet and triplet states we measure above, as well as a one-electron state in the adjacent dot, we propose a quantum dot qubit architecture that has an attractive combination of speed and fabrication simplicity. The proposed hybrid qubit consists of a double quantum dot with two electrons in one dot and one electron in the other. The qubit itself is a set of two states with total spin quantum numbers $S^2 = 3/4$ ($S = 1/2$) and $S_z = -1/2$, with the two different states being singlet and triplet in the doubly occupied dot. Gate operations can be implemented electrically, and the qubit is highly tunable, enabling fast implementation of one- and two-qubit gates in a simpler geometry and with fewer operations than in other proposed quantum dot qubit architectures with fast operations. Moreover, the system has potentially long coherence times. These are all extremely attractive properties for use in quantum information processing devices.

The hybrid qubit formed from three electrons in a double quantum dot has the potential for great speed due to presence of level crossings where the qubit becomes charge-like. Therefore, to implement a pulse-gated hybrid qubit, we first demonstrate coherent manipulations in a three-electron charge qubit.

Fast quantum oscillations of a charge qubit in a double quantum dot are demonstrated and characterized experimentally. The measured inhomogeneous dephasing time $T_2^*$ ranges
from 127 ps to \( \sim 2.1 \) ns; it depends substantially on how the energy difference of the the two qubit states varies with external voltages, consistent with a decoherence process that is dominated by detuning noise (charge noise that changes the asymmetry of the qubit’s double-well potential). In the regime with the shortest \( T_2^* \), applying a charge-echo pulse sequence increases the measured inhomogeneous decoherence time from 127 to 760 ps, demonstrating that low-frequency noise processes are an important dephasing mechanism.

Finally, we show fast coherent manipulation of three-electron states in a double quantum dot as progress towards the implementation of a pulse-gated hybrid qubit. We demonstrate that tailored pulse sequences can be used to induce coherent rotations between 3-electron quantum states. The presence of the third electron enables very fast rotations to all possible states, in contrast to the case when only two electrons are used and some rotations are slow. We show that certain pulse sequences yield coherent oscillations with a very high figure of merit (the ratio of characteristic dephasing time \( T_2^* \) to coherent oscillation period) of \( >100 \), which is a fundamental goal in the manipulation of quantum systems[1]. These oscillations can be interpreted as z-rotations of the hybrid qubit.
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I offer sincere thanks to Dr. Mark Friesen and Prof. Bob Joynt, who patiently answered my many questions in the general field of quantum computation and quantum information. I also extend my thanks to Dr. Don Savage who grows great material for us. In our group, I would like to thank Dr. Christie Simmons, a great mentor, who taught me all the measurement techniques and left me a great device to measure. She is always my idol and has greatly influenced me in my attitude toward the experiment. I would like to thank
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## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>i</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>iii</td>
</tr>
<tr>
<td>Contents</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>viii</td>
</tr>
<tr>
<td><strong>1 Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Physical Implementation of a qubit</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Thesis outline</td>
<td>5</td>
</tr>
<tr>
<td>1.3 Publications</td>
<td>6</td>
</tr>
<tr>
<td><strong>2 Tunable singlet-triplet splitting in a few-electron Si/SiGe quantum dot</strong></td>
<td>9</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>9</td>
</tr>
<tr>
<td>2.2 Experimental observation of the singlet-triplet splitting</td>
<td>10</td>
</tr>
<tr>
<td>2.3 Demonstration of tunable singlet-triplet splitting via dot translation</td>
<td>14</td>
</tr>
<tr>
<td><strong>3 Hybrid qubit: A new quantum dot spin qubit architecture</strong></td>
<td>17</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>17</td>
</tr>
<tr>
<td>3.2 Qubit design</td>
<td>18</td>
</tr>
<tr>
<td>3.3 Single qubit gate operations</td>
<td>20</td>
</tr>
<tr>
<td>3.4 Two-qubit gates</td>
<td>21</td>
</tr>
<tr>
<td>3.5 Readout and initialization</td>
<td>22</td>
</tr>
<tr>
<td>3.6 Coherence properties</td>
<td>23</td>
</tr>
</tbody>
</table>
3.7 Derivation of effective Hamiltonian ............................ 24
3.8 Connectivity between two hybrid qubits ......................... 32
3.9 CNOT gates for qubits with increased connectivities ......... 34
3.10 Details of the experimental measurements ..................... 37
3.11 Summary ................................................. 39

4 Coherent quantum oscillations and echo measurements of a Si charge
   qubit
   4.1 Introduction ............................................. 40
   4.2 Larmor oscillations (x-rotations) ............................ 41
   4.3 Ramsey fringes (z-rotations) ............................... 44
   4.4 Charge-echo experiment .................................. 46
   4.5 Measurement details and line cuts of Larmor and Ramsey oscillations ... 47
   4.6 Analysis of charge echo experiment ......................... 48
   4.7 Summary ................................................. 51

5 Fast coherent manipulation of three-electron states in a double quantum dot
   5.1 Introduction ............................................. 52
   5.2 Coherent oscillations with application of a single pulse .... 54
   5.3 Coherent oscillations with application of complex pulses ... 57
   5.4 Relationship to the quantum dot hybrid qubit ................ 59
   5.5 Simulation Method ...................................... 60
   5.6 Determination of parameters ............................... 60
   5.7 Summary ................................................. 64

6 Conclusion ................................................. 65

Bibliography ................................................. 67
List of Figures

1.1 Physical implementation of semiconductor quantum dot qubits .................. 3
1.2 Charge sensing in quantum dot qubits ................................................... 4

2.1 Scanning electron micrograph and charge stability diagram of the device ......... 10
2.2 Experimental and simulated magnitospectroscopy .................................... 13
2.3 Experimental and simulated singlet triplet splitting ................................ 14

3.1 The hybrid qubit ......................................................................................... 19
3.2 Experimental tunnel rates .......................................................................... 23
3.3 Schematic of the hybrid spin qubit ............................................................. 25
3.4 Schematic of the Heisenberg exchange couplings ....................................... 30
3.5 Schematic diagram of the (2,1) electron configurations ............................... 34
3.6 Schematic of CNOT couplings in 16 gates .................................................. 35
3.7 Schematic of CNOT couplings in 14 gates .................................................. 36
3.8 Measurement of loading and unloading rates ............................................. 38

4.1 X-rotation of the charge qubit ................................................................. 42
4.2 Z-rotation and echo experiment of the charge qubit .................................... 45
4.3 Ramsey fringe analysis ............................................................................. 48
4.4 Analysis of echo data ................................................................................. 50

5.1 Quantum oscillations between multiple energy levels I .............................. 54
5.2 Quantum oscillations between multiple energy levels II ............................. 58
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3 Simulation of oscillations with application of a single pulse using different values of $\Delta_2$</td>
<td>61</td>
</tr>
<tr>
<td>5.4 Larmor oscillations at excited anti-crossings</td>
<td>62</td>
</tr>
<tr>
<td>5.5 Simulation of oscillations with application of complex pulses using different values of $\Delta_3$</td>
<td>63</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The idea of using quantum mechanical principles to perform computational tasks is probably one of the fanciest ideas of last century. While classical computers encode one unit of information into a bit using binary numbers, either 0 or 1, and can only perform calculations on one set of numbers at one time, quantum computers encode information into a quantum bit (qubit) $|\phi\rangle$, which is an arbitrary superposition of two quantum states $|0\rangle$ and $|1\rangle$. In this case, $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$, and one seeks to perform operations on the superposition state $|\phi\rangle$. The advantage of such a computer not only lies in the fact that the qubit state $|\phi\rangle$ contains much more information than a classical bit, but also lies in the fact that performing operations on the superposition state $|\phi\rangle$ is like performing operations on the state $|0\rangle$ and $|1\rangle$ simultaneously. Operating on multiple input states simultaneously is termed “quantum parallelism” and it offers a possibility of tremendously enhanced computing power. For example, the most famous quantum algorithm, Shor’s algorithm for integer factorization, offers exponential speedup compared to the best known classical algorithms.

However, there are tradeoffs. Quantum states are fragile, in the sense that once the delicate, entangled quantum system is disturbed by its surrounding environment, it quickly loses its information to that environment. This process is usually described as decoherence. In a simplified definition of this concept, the coherence time is the characteristic time for a generic qubit state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ to be transferred to the mixture $\rho = |\alpha|^2|0\rangle \langle 0| + |\beta|^2|1\rangle \langle 1|$. 


Decoherence introduces errors in the quantum information; however, it is possible to generate quantum error-correcting codes to correct these errors, as long as the error rate is small enough. In other words, decoherence isn’t fatal if error correction can be done in time. Therefore, properly determining the coherence time of the qubit system becomes an important topic and also the goal of the project described in this thesis.

1.1 Physical Implementation of a qubit

There are many ways to physically implement qubit systems. The candidates include superconducting quantum interference devices (SQUIDs), trapped ion qubits, optics based qubits, semiconductor based qubits, to name only some. One promising scheme is semiconductor silicon quantum dot based qubits, which are promising for the following reasons:

1. The electron coherence time is longer in silicon compared to other semiconductor materials, such as GaAs, both because of its low inhomogenous nuclear field and small spin-orbit coupling.

2. Silicon processing techniques are mature in the modern semiconductor IC industry.

3. Quantum dot systems are, in principle, easy to scale up.

To implement a qubit, five criteria have to be demonstrated:

1. A scalable physical system with well characterized qubits

2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000...\rangle$

3. Long decoherence times, much longer than the gate operation time

4. A universal set of quantum gates

5. A qubit-specific measurement capability

In the silicon quantum dot system, the qubit will either be a spin qubit, which uses the spin magnetic moments of the electron as qubit states, or a charge qubit, which uses the position of the electron as qubit states. Both schemes require isolating single or few electrons in silicon. The techniques we use follow reference proposed by Loss and DiVincenzo, in which single or few electrons are confined in quantum dots.
Figure 1.1: (a) Left: TEM image of a modulation doped Si/SiGe heterostructure. Image courtesy Michelle Roberts Kelly. Right: A schematic of the heterostructure and its conduction band profile along the growth direction. Electrons are confined in the Si quantum well and form a sheet of two-dimensional electron gas (2DEG). (b) Schematic of a double quantum dot device. Metal gates are deposited on top of the heterostructure. Applying negative voltage to these gates depletes electron underneath them and isolates puddles of electrons (quantum dots). Ohmic contacts are made to the 2DEG by first depositing then annealing Au/Sb. A nearby quantum point contact (QPC) conducting channel is used as a charge sensor, by measurement of its conductance.

A modulation doped Si/SiGe heterostructure, like that shown in Fig 1.1(a), confines electrons into the Si quantum well, due to the band offset at the interface of Si and SiGe. Electrons are free to move laterally but not vertically. This layer of confined electron is called a two-dimensional electron gas (2DEG). Further confinements are realized by electrostatic gates deposited on top of the material, as in Fig 1.1(b). Negative voltages applied to these gates relative to the 2DEG locally depletes the electrons underneath the gates and creates isolated puddles of electrons in the area between. The electrons are thus confined to zero-dimensions, and the confinement area is called a quantum dot.

The quantum dot is tunnel coupled to the electron reservoir, and electrons can tunnel into or out of the quantum dot once their energy levels are appropriately set. By tuning gate voltages, one can change the energy potential of the quantum dot and transfer electrons one by one between the dot and the reservoir, making it a single electron transistor (SET). Ideally the gate voltages can be made negative enough that there is only one or a few desired electrons in the quantum dot. However, the negative voltages needed to deplete the
Figure 1.2: (a) SEM image of a typical double quantum dot device. The left quantum dot is represented by the white dashed circle. The dot is tunnel coupled with the reservoir so that electron can tunnel in and out of the dot once its energy level is aligned with the fermi level in the reservoir. The conducting channel formed by gate R and QR is called a Quantum Point Contact (QPC). It can be used as a charge sensor. (b) A typical pinch-off curve of the QPC channel. At certain gate voltage, for instance the point marked by the red cross, the conductance (current) of the channel is very sensitive to changes in the gate voltage; in other words, it is also very sensitive to the changes in the electrical environment around it. (c) Charge sensing measurement. Blue: When making gate voltage more negative, the current in the QPC decreases overall because of the direct coupling between the channel and the gate; however, at certain gate voltages, electrons are allowed to tunnel out of the quantum dot. The leaving of an electron slightly increases the conductance (current) of the channel and causes a step up in the overall decreasing current background. Grey: derivative of the blue curve. Electron tunneling signals are shown as deep dips and are clearly visible.

electrons also create more opaque tunnel barriers between the dot and the reservoir. If the tunneling channel to the reservoir is completely pinched off before most electrons can leave the dot, one will not be able to achieve a few-electron quantum dot. Therefore, there needs to be a good balance between the strength of confinement potential and tunneling barrier.

To reach the few-electron region in a quantum dot, one has to have the ability to count electrons. This is either realized by measuring the transport current through the quantum dot and observing Coulomb blockade, or by measuring the current through a nearby conducting channel whose conductance is very sensitive to the electrostatic environment around it. Such a channel, as shown in Fig 1.1(b) and Fig 1.2(a), is called a quantum point...
contact (QPC) charge sensor [4, 5, 6, 7, 8, 9, 10].

Figure 1.2(c) shows a typical measurement of QPC conductance. Since the QPC channel is capacitively coupled to other gates, when a certain gate voltage (e.g., $V_L$) is made more negative, the conductance of the channel decreases. However, when an electron tunnels in or out of the quantum dot, the electrostatic environment around the QPC channel changes, resulting in a change in the QPC conductance that is different from the change caused by direct coupling to the gates. Charge sensing measurement is better than the direct transport measurement in the sense that it is still able to identify a transition when the dot to reservoir barrier is too opaque to produce a transport current. It is also much less invasive.

1.2 Thesis outline

First, in Chapter 2, we measure the excited-state spectrum of a Si/SiGe quantum dot as a function of in-plane magnetic field and identify the spin of the lowest three eigenstates in an effective two-electron regime. We extract from the data the singlet-triplet splitting, an essential parameter for spin qubits. We find it to be tunable by lateral displacement of the dot, which is realized by changing two gate voltages on opposite sides of the device. We present calculations showing the data are consistent with a spectrum in which the first excited state of the dot is a valley-orbit state.

Next, in Chapter 3, utilizing the singlet-triplet energy splitting measured in Chapter 2, we propose a “hybrid” quantum dot qubit architecture that has an attractive combination of speed and fabrication simplicity. It consists of a double quantum dot with one electron in one dot and two electrons in the other. The qubit itself is a set of two states with total spin quantum numbers $S^2 = 3/4$ ($S = 1/2$) and $S_z = -1/2$, with the two different states being singlet and triplet in the doubly occupied dot. The architecture is relatively simple to fabricate, gate operations can be implemented electrically, and the qubit is highly tunable, enabling the electrical implementation of one- and two-qubit gates more efficiently than in other proposed qubit architectures. Moreover, the system has potentially long decoher-
ence times. These are all extremely attractive properties for use in quantum information processing devices.

In Chapter 4, we demonstrate the first step of implementing the hybrid quantum dot qubit: a three-electron charge qubit. Fast quantum oscillations of a charge qubit in a double quantum dot fabricated in a Si/SiGe heterostructure are demonstrated and characterized experimentally. The measured inhomogeneous dephasing time $T_2^*$ ranges from 127 ps to $\sim 2.1$ ns; it depends substantially on how the energy difference of the two qubit states varies with external voltages, consistent with a decoherence process that is dominated by detuning noise (charge noise that changes the asymmetry of the qubit’s double-well potential). In the regime with the shortest $T_2^*$, applying a charge-echo pulse sequence increases the measured inhomogeneous decoherence time from 127 to 760 ps, demonstrating that low-frequency noise processes are an important dephasing mechanism.

In Chapter 5, we show fast coherent manipulation of three-electron states in a double quantum dot, demonstrating progress toward implementing the hybrid qubit. We demonstrate that tailored pulse sequences can be used to induce coherent rotations between 3-electron quantum states. Some of the rotations can be interpreted as $z$-rotations of the hybrid qubit. Certain pulse sequences yield many coherent oscillations within the characteristic dephasing time $T_2^*$, giving a very high figure of merit (the ratio of coherence time to rotation time) of $> 100$. The presence of the third electron enables very fast rotations to all possible states, in contrast to the case when only two electrons are used, in which some rotations are slow. The minimum oscillation frequency we observe is $> 5$ GHz.

Finally, in Chapter 6, we offer concluding remarks.

1.3 Publications

Each of the main chapters in this thesis is based on either published or submitted work, each of which represents the work of many individuals. Here, I document my contribution to each work, as well as my collaborators.

Chapter 2 is based on Ref. [11], titled Tunable singlet-triplet splitting in a few-electron
Si/SiGe quantum dot, which I completed with Dr. Christine Simmons, Dr. Jonathan Prance, Dr. John Gamble, Dr. Mark Friesen, Dr. Don Savage, Prof. Max Lagally, Prof. Susan Coppersmith, and Prof. Mark Eriksson. Dr. Savage and Prof. Lagally grew the material, Dr. Simmons fabricated the sample, I performed the experiment, with help and assistance from Dr. Prance and Prof. Eriksson, Dr. Gamble performed the theoretical simulations with help and assistance from Dr. Friesen and Prof. Coppersmith.

Chapter 3 is based on Ref. [12], titled Fast hybrid silicon double-quantum-dot qubit, which I completed with Dr. Christine Simmons, Dr. Jonathan Prance, Dr. John Gamble, Dr. Teck Seng Koh, Dr. Yun-Pil Shim, Prof. Xuedong Hu, Dr. Don Savage, Prof. Max Lagally, Dr. Mark Friesen, Prof. Mark Eriksson and Prof. Susan Coppersmith. Prof. Coppersmith came up with the idea, Dr. Gamble, Dr. Koh performed theoretical calculations, with help from Dr. Shim, Prof. Hu and Dr. Friesen. Dr. Savage and Prof. Lagally grew the material, Dr. Simmons fabricated the sample, I performed the experiment with help from Dr. Prance and Prof. Eriksson.

Chapter 4 is based on Ref. [13], titled Coherent quantum oscillations and echo measurements of a Si charge qubit, which I completed with Dr. Christine Simmons, Dr. Dan Ward, Dr. Jonathan Prance, Robert Mohr, Dr. Teck Seng Koh, Dr. John Gamble, Xian Wu, Dr. Don Savage, Prof. Max Lagally, Dr. Mark Friesen, Prof. Susan Coppersmith and Prof. Mark Eriksson. Dr. Savage and Prof. Lagally grew the material, Dr. Simmons fabricated the sample, I performed the experiment with help from Dr. Dan Ward, Dr. Prance, Robert Mohr, Xian Wu, Prof. Coppersmith and Prof. Eriksson. I also performed simulation together with Dr. Koh with help from Dr. Gamble, Dr. Friesen and Prof. Coppersmith.

Chapter 5 is based on Ref. [14], titled Fast coherent manipulation of three-electron states in a double quantum dot, which I completed with Dr. Christine Simmons, Dr. Dan Ward, Dr. Jonathan Prance, Xian Wu, Dr. Teck Seng Koh, Dr. John Gamble, Dr. Don Savage, Prof. Max Lagally, Dr. Mark Friesen, Prof. Susan Coppersmith and Prof. Mark Eriksson. Dr. Savage and Prof. Lagally grew the material, Dr. Simmons fabricated the sample, I performed the experiment with help from Dr. Dan Ward, Dr. Prance, Xian Wu, Prof.
Coppersmith and Prof. Eriksson. I also performed simulation with help from Dr. Koh, Dr. Gamble, Dr. Friesen and Prof. Coppersmith.
Chapter 2

Tunable singlet-triplet splitting in a few-electron Si/SiGe quantum dot

2.1 Introduction

Silicon quantum dots are candidate hosts for semiconductor spin qubits, both because of long spin relaxation and coherence times for electrons in Si, and because of potential synergy with classical microelectronics. Long spin relaxation times have been demonstrated in Si quantum dots and donors [15, 16, 17, 18], and measurements of ensembles of donor-bound spins by electron spin resonance have demonstrated $T_2$ coherence times up to 2s [19]. One of the key properties of Si quantum dot spin qubits is the ability to tune in real-time tunnel rates and couplings between neighboring dots by controlling electrostatic gate voltages [20, 21, 22]. Tunable, gate-defined Si quantum dots often are designed to sit at the interface between pure Si and a barrier of either SiGe [23, 24] or SiO$_2$ [25, 26, 27].

For initialization and readout of singlet-triplet spin qubits, an essential parameter is the energy difference $E_{ST}$ between the singlet (with both electrons occupying the lowest

\footnote{This chapter is adapted from Ref. [11].}
energy single-particle state) and triplet states (built out of the lowest energy single-particle state and the first excited state) of two electrons in one dot [28, 29]. The energy $E_{ST}$ is equal to the lowest single-particle excited state energy, less a correction arising from electron-electron interactions. In Si nanostructures, which have states arising from the two low-lying valleys in the Si conduction band, the sharpness and quality of the interface between Si and the SiGe or SiO$_2$ barrier material play an important role in determining this energy [30]. Experiments have shown that quantum-confined structures can have reasonably large valley splitting, ranging from 100 $\mu$eV to 1-2 meV [31, 32, 33]. The existence of large valley splitting in Si quantum dots has led to large $E_{ST}$ and the observation of Pauli spin blockade [34, 35]. However, systematic control of the valley splitting or, more directly, $E_{ST}$ has not been demonstrated previously in a Si quantum dot.

### 2.2 Experimental observation of the singlet-triplet splitting

In this chapter, we report a magnetospectroscopy study of a Si/SiGe double quantum dot with 2 and 0 valence electrons on the left and right dots, respectively. We use a pulsed-gate voltage technique to measure the evolution of the ground and low-lying excited states
of the left dot as a function of an in-plane magnetic field $B$. We extract the magnetic field $B_{ST}$ at which the ground state changes from singlet to triplet, corresponding to the Zeeman energy equaling the singlet-triplet splitting $E_{ST}$ for the (2,0) charge configuration. We find that $B_{ST}$ is tunable by lateral displacement of the quantum dot location, achieved by simultaneously tuning voltages applied to two gates on opposite sides of the dot. $B_{ST}$ evolves systematically as a function of the gate voltages, and we measure a fractional change in $B_{ST}$ of up to 19%. Changes in gate voltages can alter both the position and shape of the electron wavefunctions in quantum dots [36, 37, 38]. Applying asymmetric changes to the voltages on either side of the quantum dot, as we do here, will change primarily the position of the quantum dot. We perform calculations showing that the fractional change in $B_{ST}$ observed is consistent with valley-orbit mixing arising from a rough Si/SiGe interface, and that a change in position alone can account for observed changes in $B_{ST}$.

A double quantum dot, shown in Fig. 2.1(a), is fabricated as described in [16]. A quantum point contact (QPC), defined by gates R and Q, is used to perform charge sensing measurements. Gate L is connected to a pulse generator (Tektronix AFG3252), allowing the application of fast voltage pulses. The dc gate voltages are tuned so that the double dot is in the few-electron regime, as shown in Fig. 2.1(b). The change in background grayscale arises from changes in the QPC sensitivity caused by capacitive cross-talk in the device [4]. Measurements are performed in a dilution refrigerator at an electron temperature $T_e = 143 \pm 10$ mK, determined as described in [16]. The electron occupation numbers are effective; we believe there are spin-zero closed shells of electrons in both the left and the right dots that do not participate in the physics discussed here.

To determine the 2-electron singlet-triplet splitting, we measure the gate voltage dependence of the transition from a single-electron spin-down state to the 2-electron ground state as a function of $B$. Figs. 2.2(a) and (b) show the transconductance $G = \partial I_{qpc}/\partial V_L$ as a function of $B$, measured with a lock-in amplifier using a 120 $\mu$V ac excitation voltage applied to gate L. The bright peak in the color plot corresponds to adding one electron to the left dot. The gate voltage of this transition first increases and then decreases as a
function of $B$.

The electrochemical potential $\mu_N$, and equivalently the gate voltage of transitions like those in Figs. 2.2(a) and (b), has a dependence on the in-plane magnetic field of the form $\partial\mu_N/\partial B = g\mu_B\Delta S_{\text{tot}}(N)$ [39]. Here $g$ is the Landé $g$-factor, $\mu_B$ is the Bohr magneton, and $\Delta S_{\text{tot}}(N)$ is the change in the $z$ component of the total spin when the $N$th electron is added to the dot. The electrochemical potential has a slope of $+g\mu_B/2$ when a spin-up electron is added (magnetic moment anti-parallel to $B$), whereas the addition of a spin-down electron results in a slope of $-g\mu_B/2$ (magnetic moment parallel to $B$). The positive slope in Fig. 2.2(a,b) at small $B$ corresponds to the addition of a spin-up electron, forming a 2-electron singlet ground state. The arrows mark $B_{\text{ST}}$, the magnetic field at which the slope changes; for $B > B_{\text{ST}}$, the added electron is spin-down, and the ground state is the triplet $T_-$. As indicated schematically in Fig. 2.2(c), the turning point of the slope corresponds to a magnetic field $B_{\text{ST}}$ at which the Zeeman energy of the state $T_-$ is equal to $E_{\text{ST}}(B = 0)$. The value of $B_{\text{ST}}$ is different in Figs. 2.2(a) and (b), indicating that $E_{\text{ST}}$ is tunable with gate voltage.

Pulsed-gate spectroscopy methods [40, 41] allow us to confirm the state identification described above, while simultaneously mapping out the excited-state energy spectrum as a function of $B$. Square wave voltage pulses of peak-to-peak amplitude 3.6 mV and frequency 50 kHz are applied to gate L, and the time-averaged value of $G$ is recorded, as shown in Fig. 2.2(d). Here, the bottom (top) line corresponds to the positive (negative) edge of the pulse bringing the 2-electron ground state into resonance with the Fermi level of the lead. Both of these lines therefore reproduce the shape of the line in Fig. 2.2(b).

The two middle lines in Fig. 2.2(d) meet at $B = 0$ and correspond to the triplet states $T_-$ and $T_0$, which are degenerate at this point, and as $B$ increases, the lines split. The $T_-$ line has a negative slope, corresponding to the addition of a spin-down electron, and this state becomes the ground state when $B = B_{\text{ST}}$. The $T_0$ line has a positive slope, corresponding to the addition of a spin-up electron. There is small chance of loading the $T_+$ after unloading the singlet into a spin-up state; however, the process is weak and produces a line at the
Figure 2.2: (a), (b) Ground state magnetospectroscopy for two different sets of gate voltages chosen so that the gate voltages for (b) favor a dot position farther to the right than those for (a) (see Fig. 2.3). The plots show the QPC conductance $G$. The arrows indicate the magnetic field $B_{ST}$ at which the Zeeman shift for the $T_-$ is equal to the zero-field $E_{ST}$. (c) Schematic diagram showing the transition as a function of $B$. (d) Excited state magnetospectroscopy using pulsed-gate voltages for the dot position corresponding to (b). (e) Simulated excited-state magnetospectroscopy for the data in panel (d).

same location as the $T_0$. 
Figure 2.3: $B_{ST}$, the magnetic field at which the ground state shifts from S to $T_-$, for different sets of gate voltages $V_L$ and $V_R$, corresponding to the symbols on the stability diagram in Fig. 1(b). Error bars are determined by the uncertainty in linear fits to lines like those in Figs. 2(a) and (b). The three gray lines show fits to three different sets of microscopic parameters that can be used to fit the experimental results.

2.3 Demonstration of tunable singlet-triplet splitting via dot translation

Fig. 2.2(e) shows a theoretical simulation of the experiment of Fig. 2.2(d), performed using a coupled rate equation model similar to that described in the supplemental material for Ref. [16]. The model includes thermal broadening but neglects energy-dependent tunneling. The $S$, $T_0$, and $T_-$ loading and unloading rates are determined by fitting the simulation to the data in Fig. 2.2(d). We find the loading rates $\Gamma_L^S = 45.1$ kHz, $\Gamma_L^{T_-} = 216$ kHz, and $\Gamma_L^{T_0} = 377$ kHz, and the unloading rates $\Gamma_U^S = 164$ kHz, $\Gamma_U^{T_-} = 354$ kHz, and $\Gamma_U^{T_0} = 183$ kHz.

Using the method illustrated in Figs. 2.2(a) and (b), we measure the transition field $B_{ST}$ at four different gate voltage configurations, corresponding to the symbols shown in Fig. 2.1(b). Along this line in the stability diagram, changes in $V_L$ and $V_R$ tend to shift the dot physically from left to right as $V_R$ ($V_L$) is made more positive (negative). As shown in Fig. 2.3, we observe a systematic increase in $B_{ST}$ as we move from left to right in the stability diagram. Over this range, $B_{ST}$ increases from $1.68 \pm 0.09$ T to $2.02 \pm 0.07$ T, a
total increase of \( \sim 19\% \).

The singlet-triplet splitting can be expressed as 
\[
E_{\text{ST}} = E_1 - E_0 + C_{01} - C_{00} + K_{\text{ST}},
\]
where 
\( E_0 \) and 
\( E_1 \) denote the ground and first excited-state energies, 
\( C_{01} \) and 
\( C_{00} \) are the Coulomb interaction energies of the two electrons in the singlet and triplet states, and 
\( K_{\text{ST}} \) is the exchange energy [42]. A simple shift of the dot position is expected to have little effect on the last three terms, which correspond to interactions between the electrons. Similarly, the shape of the wavefunction envelope should change very little as a function of the dot displacement. The variation in 
\( B_{\text{ST}} \) is not caused by a change in \( g \)-factor as a function of gate voltage, because we calculate that only 0.6% of the electron density resides in the SiGe barrier, and the \( g \)-factor changes \( \sim 2\% \) between Si and \( \text{Si}_{0.7}\text{Ge}_{0.3} \), the concentration used in our heterostructure [43]. Thus, it is important to check whether microscopic features of the quantum device can account for the changes in 
\( E_{\text{ST}} \) that we observe.

The single-particle spacing 
\[
\Delta E = E_1 - E_0
\]
has a contribution arising from the difference in valley components in the two lowest lying orbital states. The quantum well interface will have atomic steps and other sharp changes in potential that vary as a function of lateral position, and these variations can modify the coupling of the two z-valleys, contributing to a position dependence of the energy difference 
\( \Delta E \) [44, 45].

To test whether a small atomic-scale variation can account for the magnitude of the observed variations in 
\( B_{\text{ST}} \), we perform tight-binding calculations of the single particle energy levels of an electron confined near a single atomic step. The calculations use a two-dimensional tight-binding Hamiltonian similar to Refs. [46, 47], including a parabolic lateral confinement potential. The fitting procedure varies the position of the atomic step, the confinement length scale, and the vertical electric field, enabling a calculation of the variation in 
\( \Delta E \) as a function of gate voltage. To compare with the measured 
\( B_{\text{ST}} \), we also fit the sum of the Coulomb and exchange energies 
\( (C_{01} - C_{00} + K_{\text{ST}}) \), and the results are plotted in Fig. [2.3]. The fitting is underconstrained, as there are many physical ways to produce similar valley splitting. To indicate the types of variations possible, three results are plotted in Fig. [2.3] as the solid, dashed, and dotted lines, and all three calculations can reproduce
the magnitude of the observed change in $B_{ST}$. The lowest excited-state can be classified as “orbital-like” when the calculated wavefunction contains a lateral node or “valley-like” when it does not, and both cases occur. For valley-like excitations, lateral translation of the dot with respect to a step results in a tunable valley splitting. For orbital-like excitations, strong valley-orbit coupling enables a tunable orbital energy splitting. Thus, atomic-scale structure of the quantum well interface is sufficient to produce valley-orbit mixing large enough to account for the experimental observations, even under the assumption that the dot shape is unchanged as a function of gate voltage.
Chapter 3

Hybrid qubit: A new quantum dot spin qubit architecture

3.1 Introduction

Tremendous progress towards the development of working electrically-gated quantum dot qubits has been made over the past decade, and single-qubit operations have been demonstrated for logical qubits implemented in single [48], double [29], and triple [49] quantum dots in GaAs heterostructures. However, even with sophisticated pulse sequences that lead to coherence times up to 200 µs [50], the important figure of merit, the number of gate operations that can be performed within the qubit coherence time [51 52 53], needs to be improved significantly for quantum dot qubits to become useful. Moreover, it is highly desirable that a given implementation be as simple as possible.

In this chapter, we present a relatively simple double-dot qubit architecture in which a universal set of fast gate operations can be implemented. Each qubit consists of a double quantum dot with two electrons in one dot and one electron in the other. The qubit itself is the set of two low-lying electronic states with total spin quantum numbers $S = 1/2$ (square of the total spin of $3\hbar^2/4$) and $S_z = -1/2$ (z-component of total spin of $-\hbar/2$). These states

---

1^This chapter is adapted from Ref. [12].
form a decoherence-free subspace that is insensitive to long-wavelength magnetic flux noise; moreover, decoherence processes that do not explicitly couple to spin or induce a transition of an electron to the reservoir do not induce transitions that go outside of the subspace of an individual qubit [54].

The gate operations are all implemented using purely electrical manipulations, enabling much faster gates than using ac magnetic fields [3, 48], inhomogeneous dc magnetic fields [55, 29, 50], or mechanisms using spin-orbit coupling [56, 57]. The qubit has the same symmetries in spin space as the triple-dot qubit proposed by DiVincenzo et al. [58], but is simpler to fabricate because it requires a double dot instead of a triple dot. The hybrid qubit proposed here also has significant advantages over the three-dot qubit for implementing multi-qubit operations: two hybrid qubits made of four dots in a linear array have higher effective connectivity than the similar linear array of dots considered in Ref. [58]. We show that this increased effective connectivity can reduce the number of manipulations required to implement two-qubit gates.

We present evidence that implementing this qubit in silicon is feasible. The development of silicon qubits has attracted substantial interest [60, 15, 18, 16], because spins in silicon have longer coherence times than spins in many other semiconductors, because of both the weak spin-orbit coupling and the low nuclear spin density in silicon [61, 62, 19]. Here, we measure a triplet-singlet relaxation time in a single silicon dot to be $> 100$ ms and demonstrate readout of the singlet and triplet states of two electrons in a silicon dot. We estimate dephasing times theoretically to be on the order of microseconds, long enough to achieve high fidelity quantum operations.

### 3.2 Qubit design

An important advantage of the qubit proposed here is that all qubit manipulations can be implemented using electric and not magnetic fields, resulting in fast operations [58]. To understand why electrical manipulation of our qubit is possible, we enumerate the possible transitions between spin states of three electrons that can be induced by spin-conserving
matrix elements that shift a single electron from the singlet or triplet state in the left dot to the lowest energy state in the right dot. Fast qubit gate operations are performed by applying gate voltages that change the energy splittings between the singlet and triplet states in the left dot and that change the tunnel couplings $g_S$ and $g_T$ between the two dots. (a) Introducing tunneling between the dots induces transitions between $|0\rangle_L$ and $|1\rangle_L$. Starting from $|0\rangle_L$, in which the electrons in the left dot are in a singlet, if an electron tunnels from the left dot to the right dot, and then the other electron tunnels back to the left dot, the spins in the left dot will end up in a triplet. The actual process conserves the total spin quantum numbers $S^2$ and $S_z$ and yields transitions between $|0\rangle_L$ and $|1\rangle_L$ (see Section 3.7). (b) and (c): Schematic illustrating independent tuning of the coupling between the electron in the singly occupied dot and the singlet and triplet states in the doubly occupied dot via the barrier height and relative energies in the two dots, as described in the text. (d): Effective connectivity of two hybrid qubits composed of four dots in a linear geometry. Each connection is a tunable two-electron interaction. There are eight effective connections, compared to five effective connections in a linear array of six dots for the qubits considered in Ref. 58, shown in (e). For (d), a two-qubit gate equivalent to CNOT up to local (one-qubit) unitary operations can be implemented in 16 steps, compared to 18 for (e) 59 (see Section 3.2). (f): Connectivity for which a fourteen-operation two-qubit gate equivalent to CNOT up to local unitary operations has been found (see Section 3.2).

Figure 3.1: The logical qubit states of the hybrid qubit are $|0\rangle_L = |S\rangle|\downarrow\rangle$ and $|1\rangle_L = \sqrt{\frac{1}{3}}|T_0\rangle|\downarrow\rangle - \sqrt{\frac{2}{3}}|T_-\rangle|\uparrow\rangle$, where $|S\rangle$, $|T_-\rangle$, and $|T_0\rangle$ are two-particle singlet ($S$) and triplet ($T$) states in the left dot, and $|\uparrow\rangle$ and $|\downarrow\rangle$ respectively denote a spin-up and spin-down electron in the right dot. Fast qubit gate operations are performed by applying gate voltages that change the energy splittings between the singlet and triplet states in the left dot and that change the tunnel couplings $g_S$ and $g_T$ between the two dots. Starting from $|0\rangle_L$, in which the electrons in the left dot are in a singlet, if an electron tunnels from the left dot to the right dot, and then the other electron tunnels back to the left dot, the spins in the left dot will end up in a triplet. The actual process conserves the total spin quantum numbers $S^2$ and $S_z$ and yields transitions between $|0\rangle_L$ and $|1\rangle_L$ (see Section 3.7). (b) and (c): Schematic illustrating independent tuning of the coupling between the electron in the singly occupied dot and the singlet and triplet states in the doubly occupied dot via the barrier height and relative energies in the two dots, as described in the text. (d): Effective connectivity of two hybrid qubits composed of four dots in a linear geometry. Each connection is a tunable two-electron interaction. There are eight effective connections, compared to five effective connections in a linear array of six dots for the qubits considered in Ref. 58, shown in (e). For (d), a two-qubit gate equivalent to CNOT up to local (one-qubit) unitary operations can be implemented in 16 steps, compared to 18 for (e) 59 (see Section 3.2). (f): Connectivity for which a fourteen-operation two-qubit gate equivalent to CNOT up to local unitary operations has been found (see Section 3.2).

manipulations. When three spin-1/2 entities are added, the resulting 8 total spin eigenstates form a quadruplet with $S = 3/2$ and $S_z = 3/2, 1/2, -1/2, -3/2$, and two doublets, each with $S = 1/2, S_z = \pm 1/2$, where the square of total spin is $\hbar^2 S(S+1)$ and the z-component of the total spin is $\hbar S_z$. Only states with the same $S$ and $S_z$ can be coupled by spin-independent terms in the Hamiltonian. We choose to use the group of two states with $S = 1/2, S_z = -1/2$ for the states of the qubit.

As discussed in Ref. 58, the two states of the logical qubit with $S = 1/2$ and $S_z = -1/2$
can be written as

\begin{align}
|0\rangle_L &= |S\rangle|\downarrow\rangle, \\
|1\rangle_L &= \sqrt{\frac{1}{3}}|T_0\rangle|\downarrow\rangle - \sqrt{\frac{2}{3}}|T_-\rangle|\uparrow\rangle.
\end{align}

(3.1) (3.2)

In our case, $|S\rangle$ is the singlet ($\frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$), $T_0$ triplet ($\frac{(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)}{\sqrt{2}}$), and $T_-$ triplet ($|\downarrow\downarrow\rangle$) in the left dot, and $|\uparrow\rangle$ and $|\downarrow\rangle$ respectively denote a spin-up and spin-down electron in the right dot. The essential difference between our system and that of [58] is that the singlet and triplet states are of two electrons in one dot instead of two different dots, as depicted in Fig. 3.1.

### 3.3 Single qubit gate operations

We now discuss how gate operations are implemented in this qubit in terms of the elementary operations implemented by changes of gate voltages in the device. A complete set of single-qubit manipulations consists of one that changes the energy splitting between the qubit states and another that drives transitions between the qubit states. The energy difference between the two qubit states is mainly the singlet-triplet splitting in the doubly occupied dot, and this splitting indeed can be tuned by changing gate voltages in both GaAs/GaAlAs [63] and in Si/SiGe dots [11]. In Si/SiGe systems, changing the voltage on a global top-gate should also change the singlet-triplet splitting [64, 65].

Transitions between the two states of the hybrid qubit can be induced by changing the off-diagonal terms in the reduced Hamiltonian. These terms are each proportional to $g_i^2/\Delta E_i$, where $g_i$ is the relevant tunneling amplitude and $\Delta E_i$ is the energy difference between the relevant state with two electrons on the left dot and the virtual state in which an electron has tunneled from state $i$ in the left dot onto the right dot. Explicit calculations of the effective spin Hamiltonian obtained by a canonical transformation that systematically eliminates higher energy states [66, 67, 68] demonstrate that increasing the tunnel couplings between the quantum dots indeed drives transitions between the two states of the qubit (see Section 3.7). Therefore, gate modulations that change the $g_i$ will induce transitions
between the qubit states, and modulations of the energy difference $\Delta E_i$ will similarly induce transitions when the $g_i$ are non-negligible. We note that when the singlet-triplet splitting $E_{ST}$ is nonzero, Rabi flops are performed by modulating the off-diagonal terms at the angular frequency $\Omega$ satisfying $\hbar \Omega = E_{ST}$. This modulation is easier to achieve experimentally when $E_{ST}$ is not too large. A singlet-triplet splitting of 0.05 meV, typical of splittings measured in quantum dots fabricated in Si/SiGe heterostructures [31, 32], corresponds to a frequency of $\sim 10$ GHz. Quantum dot gate operations have already been achieved at this speed [69], and efficient schemes exist for refocusing the fast rotations [70].

While the two manipulations obtained by changing the singlet-triplet splitting in one dot or the tunnel coupling between two dots described above are sufficient for achieving arbitrary single qubit gates, a larger set of elementary operations (or, equivalently, more fine-grained control of the terms in the effective Hamiltonian) is useful because it enables two-qubit gates to be implemented with fewer elementary operations. We note that $g_S$ and $g_T$, the tunneling matrix elements that shift a single electron from the singlet or triplet state in the left dot to the lowest energy state in the right dot, can be tuned separately. Decreasing the tunnel barrier, as shown in Fig. 3.1(c), increases both tunnel rates, whereas changing the difference between the overall energies in the left and right dots, as in Fig. 3.1(b), can change the ratio of the two tunnel rates, because of energy-dependent tunneling [71, 72].

The tunable degrees of freedom (the singlet-triplet splitting and the tunnel rates into the singlet and into the triplet) are denoted schematically in Fig. 3.1(d) as dashed lines.

### 3.4 Two-qubit gates

The spin symmetries of the hybrid qubit are the same as in the three-dot qubit of Ref. [58], and two-qubit gates are implemented similarly; however, because the hybrid qubit has higher effective connectivity, two-qubit gates can be implemented with fewer elementary operations. The increased connectivity for dots in a linear array is illustrated schematically in Fig. 3.1(d-f). Fig. 3.1(d) shows two hybrid qubits with eight effective connections, while Fig. 3.1(e) shows the five effective connections of two triple-dots in a linear array.
Fig. 3.1(f) shows a different arrangement of two double-dots, also with eight effective connections (section 3.8 shows how the hybrid qubit can achieve these connections). We have found sequences, presented in section 3.8, of 16 and 14 two-qubit operations that yield gates equivalent to CNOT up to local unitary operations for Fig. 3.1(d) and Fig. 3.1(f), respectively. In comparison, 18 operations are needed for Fig. 3.1(e) [59]. These shorter gate sequences provide strong evidence that increased effective connectivity can enable implementation of gates of two logical qubits with fewer elementary two-qubit operations.

3.5 Readout and initialization

Readout of the qubit states can be performed by exploiting the difference in tunnel rates coupling the singlet and triplet states of the doubly occupied dot to the lead [73].

Fig. 3.2 shows charge sensing measurements of a Si/SiGe double quantum dot in which the ground state of the left dot when occupied by two electrons at low field is a singlet S, whereas at large magnetic field $B$ the ground state is the triplet $T_-$. (All electron numbers refer to the effective electron number; the actual number may include a spin-zero closed shell of electrons in addition to the valence electrons we study here. Details of the measurements are presented in section 3.10.) To measure the tunnel rate into the dot, we start with one electron in the left dot. We apply a step increase in voltage to gate PL, causing an electron to tunnel from the lead into the dot, and changing the dot occupation from one to two. Fig. 3.2(a) shows an average of 300 measurements of the charge sensor current in response to the electron tunneling into the dot for $B = 1$ T and $B = 3$ T, showing exponential decays corresponding to loading a single electron with tunneling rates $\gamma_{\text{load}} = 81$ Hz ($B = 1$ T, ground state $S$) and $\gamma_{\text{load}} = 521$ Hz ($B = 3$ T, ground state $T_-$). (See Section 3.10). Fig. 3.2(c) shows analogous measurements of electrons tunneling out of the dot that yield tunnel rates $\gamma_{\text{unload}} = 182$ Hz (ground state $S$) and $\gamma_{\text{unload}} = 645$ Hz (ground state $T_-$). The difference in tunnel rates between the singlet and triplet states is thus large and suitable for both readout and initialization.
Coherence properties

While the spin symmetries of the hybrid qubit are identical to those of the three-dot qubit in Ref. [58], the coherence properties are different because the singlet and triplet states have different spatial wavefunctions. An essential component of this qubit is a long lifetime for the triplet state of the dot with two electrons. We measure this lifetime by applying a step in voltage to gate PL large enough that both the ground state singlet $S$ and the triplet $T_-$ excited state are energetically accessible at magnetic field $B = 1.5$ T. States $S$ and $T_-$ each load with some probability. After a dwell time $\Delta t$, the voltage is returned to its previous value, and an electron tunnels out of the dot. As $\Delta t$ is increased, the probability that the
electrons remain in the \( T_- \) state decreases exponentially with a characteristic time \( T_1 \), and the tunnel rate to the lead decays with the same characteristic time. Fig. 3.2(d) shows the result of such a measurement; it yields \( T_1 = 141 \pm 12 \) ms. This slow relaxation time is significantly longer than the value of \( \sim 3 \) ms measured in GaAs [73] and is consistent with theoretical estimates that account for Rashba spin-orbit coupling and phonon-assisted hyperfine coupling [74, 75].

The different charge distributions of the two qubit states gives rise to dephasing due to electron-phonon coupling [76, 77] and charge noise [78]. Our calculations indicate that for realistic states, the intervalley component of the electron-phonon dephasing term is the most important, and leads to \( T_2 \sim 1 \) \( \mu \)s [79]. Dephasing due to charge noise is suppressed in the hybrid qubit compared to charge qubits [80] because the changes in charge distributions are confined to a single quantum dot, making the effective dipole moment much smaller (indeed, the dipole moment vanishes in the limit of harmonic dot potentials) [79]. Therefore, charge fluctuation-induced decoherence is greatly suppressed in the hybrid qubit compared to double dot charge qubits. Simple estimates indicate that decoherence rates induced by nuclear spins will be similar to those in singlet-triplet qubits [50].

### 3.7 Derivation of effective Hamiltonian

This section presents the derivation of an effective Hamiltonian that describes the quantum behavior of the hybrid double quantum dot spin qubit in the regime of low energy excitations. Gate voltages control the detuning and the tunnel barrier of the dots. We will assume that the gate voltages applied to the quantum dots are kept in the regime in which the the (2,1) charge configuration has lowest energy. Fig. 3.3 shows the detuned double quantum dot system with the lowest single particle energies of each dot labeled by 1 and 2.

Before presenting the details of our calculations, we recall the cartoon presented in Fig. 3.1(a) that illustrates why introducing tunneling between dots induces transitions between the singlet and triplet states of the doubly occupied dot. When tunneling is allowed, there is a matrix element between the (2,1) state with the two electrons in the left dot in
Figure 3.3: Schematic of the hybrid spin qubit. Here, levels L1 and L2 refer to the lowest single particle energies of the left, doubly occupied dot and R1 is the lowest single particle energy of the right, singly occupied dot. We assume that changing appropriate gate voltages changes the detuning, or relative energies of the states in the two dots (though always with the (2,1) charge configurations having the lowest energy), and that gates can be used to change the tunnel barrier between the quantum dots.

a singlet and the virtual (1,2) state in which the two electrons in the right dot are in a singlet, which in turn is coupled to the (2,1) state with the two electrons in a triplet state. The coupling between the (2,1) singlet and (2,1) triplet states is of order $g^2/\Delta E$, where $g$ is the tunnel coupling and $\Delta E$ is the energy difference between the (2,1) state and the (1,2) excited state.

Here we provide a more detailed calculation of the couplings between the states of the hybrid qubit. The Hamiltonian describing interacting electrons, in general, can be written in the Hubbard-like form

$$\hat{H} = \sum_{\alpha,i,s} (\epsilon_{\alpha i} + \mu_\alpha) \hat{n}_{\alpha i s} + \sum_{\alpha \neq \beta} \sum_{i,j} \sum_s g_{\alpha i,\beta j} \hat{c}_{\alpha i s}^\dagger \hat{c}_{\beta j s} + \frac{1}{2} \sum_{\alpha,\beta,\gamma,\delta} \sum_{i,j,k,l} \sum_{s,s'} \sum_{\alpha,i,s} \Lambda_{\alpha \beta \gamma \delta}^{\alpha \beta \gamma \delta} \hat{c}_{\alpha i s}^\dagger \hat{c}_{\beta j s'}^\dagger \hat{c}_{\gamma k s'} \hat{c}_{\delta l s},$$

(3.3)

where we have used Greek indices $\alpha, \beta, \gamma, \delta$ to refer to the quantum dots, Roman indices $i, j, k, l$ to refer to the orbitals of a quantum dot, and $s, s'$ are spin indices. In this notation, operator $\hat{c}_{\alpha i s}^\dagger$ ($\hat{c}_{\alpha i s}$) creates (annihilates) an electron with spin $s$ in the $i$-th orbital of the $\alpha$-th quantum dot, and $\hat{n}_{\alpha i s} = \hat{c}_{\alpha i s}^\dagger \hat{c}_{\alpha i s}$ is the number operator.

In the Hamiltonian of Eq. 3.3, the single particle energy of the $i$th orbital in the $\alpha$th quantum dot is given by $\epsilon_{\alpha i}$, and the energy shift of the quantum dot is given by $\mu_\alpha$. This energy shift can be controlled experimentally by application of some gate voltage. The
second term on the right-hand side of Eq. 3.3 is the tunneling operator, which describes the
tunneling of an electron with spin \(s\) between the \(j\)-th orbital of the \(\beta\)-th quantum dot and the
\(i\)-th orbital of the \(\alpha\)-th quantum dot, where \(\alpha\) and \(\beta\) refer to different quantum dots. Orbital
and tunneling energies can be calculated by single particle integrals between quantum dot
orbital wavefunctions \(\phi_{\alpha i}(r)\),

\[
e_{\alpha i} = \int dr \phi_{\alpha i}^*(r) \left[ \frac{1}{2m^*} \hat{p}^2 + V_p(r) \right] \phi_{\alpha i}(r), \tag{3.4}
\]

\[
g_{\alpha i,\beta j} = \int dr \phi_{\alpha i}^*(r) \left[ \frac{1}{2m^*} \hat{p}^2 + V_p(r) \right] \phi_{\beta j}(r), \tag{3.5}
\]

where \(V_p(r)\) is the model quantum dot confinement potential, \(m^*\) is the effective mass of an
electron in the conduction band, \(q\) is the electronic charge and \(\hat{p}\) is the momentum operator.

The prefactor of the third term in the Hamiltonian in Eq. 3.3 is a two-electron Coulomb
integral between quantum dot orbital wavefunctions \(\phi(r)\), given by

\[
\Lambda_{ijkl}^{\alpha\beta\gamma\delta} = \int\int dr\, dr' \phi_{\alpha i}^*(r) \phi_{\alpha j}^*(r') \frac{q^2}{4\pi\epsilon_r\epsilon_0 |r - r'|} \phi_{\gamma k}(r') \phi_{\delta l}(r), \tag{3.6}
\]

where \(\epsilon_0\) is the permittivity of free space and \(\epsilon_r\) the relative permittivity of the material.

When the quantum dot indices refer to the same dot \((\alpha = \beta = \gamma = \delta)\), we get the
intra-dot Coulomb energies. This can be further categorized into the direct and exchange
Coulomb energies. When the orbital indices \(i = l\) and \(j = k\), we obtain the direct Coulomb
energy, \(\Lambda_{ijji}^{\alpha\alpha\alpha\alpha} \equiv C_{\alpha i,\alpha j}\). When the orbital indices \(i = k\) and \(j = l\), we get the exchange
Coulomb energy, \(\Lambda_{ijij}^{\alpha\alpha\alpha\alpha} \equiv K_{\alpha i,\alpha j}\). When the quantum dot indices refer to the different dots
\((\alpha = \delta, \beta = \gamma, \alpha \neq \beta)\), we obtain the direct and exchange Coulomb energies between the
electrons localized in the different quantum dots, i.e. \(\Lambda_{ijij}^{\alpha\beta\beta\alpha} \equiv C_{\alpha i,\beta j}\) and \(\Lambda_{ijij}^{\alpha\beta\alpha\beta} \equiv K_{\alpha i,\beta j}\).

These direct and exchange Coulomb energies are given by

\[
C_{\alpha i,\beta j} = \int\int dr\, dr' \phi_{\alpha i}^*(r) \phi_{\alpha j}^*(r') \frac{q^2}{4\pi\epsilon_r\epsilon_0 |r - r'|} \phi_{\beta j}(r') \phi_{\alpha i}(r), \tag{3.7}
\]

\[
K_{\alpha i,\beta j} = \int\int dr\, dr' \phi_{\alpha i}^*(r) \phi_{\alpha j}^*(r') \frac{q^2}{4\pi\epsilon_r\epsilon_0 |r - r'|} \phi_{\beta i}(r') \phi_{\alpha j}(r). \tag{3.8}
\]

Because of the non-vanishing overlap of the orbital wavefunctions between different
quantum dots in the regime where the dots are coupled, other integrals also yield nonzero
contributions. In general, for a system of two quantum dots, the integrals $\Lambda_{ijkl}^{\alpha\beta\alpha\beta}$ and $\Lambda_{ijkl}^{\beta\alpha\beta\alpha}$ may be non-zero for different orbitals $i,j,k$ and $l$, due to the overlap between orbitals centered on different quantum dots $\alpha$ and $\beta$. On the other hand, for a single quantum dot, if we assume a symmetric confinement potential $V_p(r)$, such terms vanish.

By categorizing these energies into intra-dot, inter-dot and tunneling energies, we can transform the Hamiltonian of Eq. 3.3 into a more familiar form \cite{81} with intra-dot energy, inter-dot energy and tunneling operators $\hat{U}_0$, $\hat{U}_1$ and $\hat{T}$, respectively:

$$\hat{H} = \hat{U}_0 + \hat{U}_1 + \hat{T}, \quad (3.9)$$

where

$$\hat{U}_0 = \sum_{\alpha,i,s} (e_{\alpha i} + \mu_{\alpha}) \hat{n}_{\alpha i s} + \frac{1}{2} \sum_{\alpha} \sum_{i, j} \sum_{s, s'} \left( C_{\alpha i, \alpha j} \hat{c}_{\alpha i s}^\dagger \hat{c}_{\alpha j s'}^\dagger \hat{c}_{\alpha j s'} \hat{c}_{\alpha i s} + K_{\alpha i, \alpha j} \hat{c}_{\alpha i s}^\dagger \hat{c}_{\alpha j s'}^\dagger \hat{c}_{\alpha i s} \hat{c}_{\alpha j s'} \right), \quad (3.10)$$

$$\hat{U}_1 = \frac{1}{2} \sum_{\alpha \neq \beta} \sum_{\gamma \neq \delta} \sum_{i, j, k, l} \sum_{s, s'} \Lambda_{ijkl}^{\alpha\beta\gamma\delta} \hat{c}_{\alpha i s}^\dagger \hat{c}_{\beta j s'}^\dagger \hat{c}_{\gamma k s'} \hat{c}_{\delta l s}, \quad (3.11)$$

$$\hat{T} = \sum_{\alpha \neq \beta} \sum_{i, j} \sum_s \hat{g}_{\alpha i, \beta j} \hat{c}_{\alpha i s}^\dagger \hat{c}_{\beta j s}. \quad (3.12)$$

The Hilbert space of a system of three electrons confined in a double quantum dot potential spans the space of states with charge configurations $(3,0)$, $(0,3)$, $(2,1)$ and $(1,2)$. Here, number pairs $(n,m)$ denote the number of electrons in the left $(n)$ and right $(m)$ dots. We shall denote by $A^{(2,1)}$, $A^{(1,2)}$, $A^{(3,0)}$ and $A^{(0,3)}$ the set of states spanning the subspaces of $(2,1)$, $(1,2)$, $(3,0)$ and $(0,3)$ states respectively. The Hilbert space of the hybrid qubit is given by direct sum of these subspaces, and will be denoted by $A \equiv A^{(2,1)} \oplus A^{(1,2)} \oplus A^{(3,0)} \oplus A^{(0,3)}$. In the Hilbert space $A$, the inter-dot tunneling operator $\hat{T}$ describes the coherent tunneling of electrons between states of the distinct subspaces whereas the intra-dot and inter-dot energy operators $\hat{U}_0$ and $\hat{U}_1$ do not couple these subspaces.

We can treat the tunneling matrix elements perturbatively and block-diagonalize the Hamiltonian by using the Schrieffer-Wolff transformation \cite{66, 67, 68}. In this way we can transform the Hubbard-like Hamiltonian into a spin Hamiltonian $\hat{H}'$ by a unitary transfor-
\[
\hat{H}' = e^{i\hat{S}} \hat{H} e^{-i\hat{S}}
= \hat{H} + [i\hat{S}, \hat{H}] + \ldots
= (\hat{U}_0 + \hat{U}_1) + \hat{T} + [i\hat{S}, (\hat{U}_0 + \hat{U}_1)] + [i\hat{S}, \hat{T}] + \ldots
\] (3.13)

We note that \(i\hat{S}\) can be chosen such that \(\hat{T} + [i\hat{S}, (\hat{U}_0 + \hat{U}_1)] = 0\), yielding an effective Hamiltonian for states with a fixed number of electrons on each dot. The transformed Hamiltonian becomes, in the lowest order, \(\hat{H}' \approx \hat{U}_0 + \hat{U}_1 + [i\hat{S}, \hat{T}]\). Ref. [67] provides an explicit expression for the choice of \(i\hat{S}\), given by

\[
i\hat{S} = \sum_{n,m} |\varphi_n\rangle \frac{\langle \varphi_n | \hat{T} | \varphi_m \rangle}{\langle \varphi_n | \hat{U} | \varphi_n \rangle - \langle \varphi_m | \hat{U} | \varphi_m \rangle} \langle \varphi_m |,
\] (3.14)

where the states \(|\varphi_n\rangle \in \mathcal{A}\) are eigenstates of \(\hat{U} \equiv \hat{U}_0 + \hat{U}_1\). Since \(|\varphi_n\rangle\) are eigenstates of \(\hat{U}\), from the eigenvalue equation, \(\hat{U} |\varphi_n\rangle = U_n |\varphi_n\rangle\), the orthogonality condition \(\langle \varphi_n | \varphi_m \rangle = \delta_{nm}\), and the completeness relation, \(\sum_n |\varphi_n\rangle \langle \varphi_n| = \hat{1}\), one obtains

\[
[i\hat{S}, \hat{U}] = -\hat{\mathcal{T}}.
\] (3.15)

The first order commutator \([i\hat{S}, \hat{T}]\) gives

\[
\langle \varphi_k | [i\hat{S}, \hat{T}] | \varphi_{k'} \rangle = \sum_m \frac{\langle \varphi_k | \hat{T} | \varphi_m \rangle \langle \varphi_m | \hat{T} | \varphi_{k'} \rangle}{U_k - U_m} - \sum_m \frac{\langle \varphi_k | \hat{T} | \varphi_m \rangle \langle \varphi_m | \hat{T} | \varphi_{k'} \rangle}{U_m - U_{k'}}.
\] (3.16)

Therefore, the first order commutator contains terms describing the second order hopping of an electron between states in the subspaces. These second order processes produce hoppings between the subspaces as follows: \(\mathcal{A}^{(0,3)} \rightarrow \mathcal{A}^{(1,2)} \rightarrow \mathcal{A}^{(0,3)}\), \(\mathcal{A}^{(3,0)} \rightarrow \mathcal{A}^{(2,1)} \rightarrow \mathcal{A}^{(3,0)}\), \(\mathcal{A}^{(1,2)} \rightarrow \mathcal{A}^{(0,3)} \rightarrow \mathcal{A}^{(1,2)}\), \(\mathcal{A}^{(2,1)} \rightarrow \mathcal{A}^{(3,0)} \rightarrow \mathcal{A}^{(2,1)}\) and \(\mathcal{A}^{(2,1)} \rightarrow \mathcal{A}^{(1,2)} \rightarrow \mathcal{A}^{(2,1)}\).

Eq. [3.16] shows that second order processes have coefficients that are are inversely proportional to the magnitude of the intra-dot energy difference \(|U_m - U_k|\) between states \(|\varphi_m\rangle\) and \(|\varphi_k\rangle\). Since intra-dot energies of the states in subspaces \(\mathcal{A}^{(0,3)}\) and \(\mathcal{A}^{(3,0)}\) are larger than
those of the states in subspaces $A^{(2,1)}$ and $A^{(1,2)}$ because of the much larger Coulomb repulsion between electrons in triply-occupied quantum dots, second order hopping processes involving triply-occupied states are virtual processes with much smaller probability compared to hoppings between states in the subspaces $A^{(2,1)}$ and $A^{(1,2)}$. As we are interested in the low-lying energy states of the system, we can restrict our basis to the states in the subspace of $A^{(2,1)} \oplus A^{(1,2)}$ and ignore triply-occupied states.

Because the qubit states are encoded in the $S = \frac{1}{2}, S_z = -\frac{1}{2}$ two-level subspace belonging to $A^{(2,1)}$, we can preserve the (2,1) charge configuration of the qubit by adjusting the gate voltages that define the dots so that the lowest energy $A^{(1,2)}$ configuration has higher energy than both of the qubit states in $A^{(2,1)}$. The energy coefficient of the $n$-th order of the commutators in Eq. 3.13 is proportional to $\frac{g^{n+1}}{(U_m - U_k)^n}$ and the series in Eq. 3.13 can be truncated to desired order when $g \ll |U_m - U_k|$. Here, we keep only the lowest order contribution.

The effective Hamiltonian that describes the low-lying energy states can then be projected onto the subspace $A^{(2,1)}$ by the projection operator $\hat{P}^{(2,1)} \equiv \sum_k |\phi_k^{(2,1)}\rangle \langle \phi_k^{(2,1)}|$, where the set of states $\{|\phi_k^{(2,1)}\rangle\}$ span $A^{(2,1)}$. The effective Hamiltonian is given by

$$\hat{H}_{\text{eff}} = \hat{P}^{(2,1)} \hat{H}^{(2,1)} \hat{P}^{(2,1)},$$

(3.17)

With the understanding that we are restricting our basis to the $A^{(2,1)}$ subspace, we drop the projection operators $\hat{P}^{(2,1)}$. Using the bilinear spin identity $\vec{S}_i = \frac{1}{2} \sum_{s,s'} \hat{c}_{is}\vec{\sigma}_{ss'}\hat{c}_{is'}$, we cast the effective Hamiltonian into the form

$$\hat{H}_{\text{eff}}^{(t-J)} = \hat{U} + \hat{H}_J + \hat{H}_{J'},$$

(3.18)

reminiscent of the t-J model [82]. The terms $\hat{H}_J$ and $\hat{H}_{J'}$ are in the form of inter-dot and
intra-dot Heisenberg-like exchange interactions, and are given explicitly by

\[
\hat{H}_J = \frac{1}{2} \sum_i J_i \left[ \left( \sum_{s,s'} c_{Lis}^\dagger \sigma_{ss'} c_{Lis'} \right) \cdot \left( \sum_{s,s'} c_{Ris}^\dagger \sigma_{ss'} c_{Ris'} \right) - \hat{n}_{Li}\hat{n}_{R1} \right]
\]

\[
= \frac{1}{2} \sum_i J_i (\hat{\mathbf{S}}_{Li} \cdot \mathbf{S}_{R1} - \hat{n}_{Li}\hat{n}_{R1}), \tag{3.19}
\]

\[
\hat{H}_{J'} = \frac{1}{2} \sum_{i \neq j} J'_{ij} \left[ \left( \sum_{s,s'} c_{Lis}^\dagger \sigma_{ss'} c_{Ljs} \right) \cdot \left( \sum_{s,s'} c_{Ris}^\dagger \sigma_{ss'} c_{Rjs} \right) \right.

\left. - \left( \sum_s c_{Lis}^\dagger c_{Ljs} \right) \hat{n}_{R1} \right] \tag{3.20}
\]

Labels L and R denote the left and right quantum dots and \(i,j = 1, 2\) refer to the lowest two orbitals of a quantum dot. The meaning of these operators are as follows. \(\hat{H}_J\) is the Heisenberg exchange coupling between electron spin in the \(i^{th}\) orbital of the left quantum dot and the electron spin in the ground orbital of the right quantum dot (R1). \(\hat{H}_{J'}\) is the exchange coupling between the electron spins in the \(i^{th}\) and \(j^{th}\) orbital of the left quantum dot, mediated by the spin in the ground orbital of the right quantum dot.

Here, we note that the difference in the energy eigenvalues \(U_m\) and \(U_k\), of the total intra- and inter-dot energy operator \(\hat{U} \equiv \hat{U}_0 + \hat{U}_1\) in the subspaces of \(A^{(2,1)}\) and \(A^{(1,2)}\)
appear in the denominator of each of the terms in Eq. 3.16. If \( m \) and \( k \) refer to the ground eigenstates of these subspaces, the biggest contribution to the energy difference in the denominator would be due to the detuning or energy shift differences between the dots, followed contributions from differences in intra-dot energies, and finally, from differences in inter-dot energies in states \( m \) and \( k \). Therefore, one could omit the inter-dot energy operator \( \hat{U}_1 \) from the Hubbard-like Hamiltonian (Eq. 3.9) from the start, without changing the physical processes described by the t-J Hamiltonian (Eq. 3.18) as it contributes only a small correction to the Heisenberg coupling constants \( J_i \) and \( J'_{ij} \). In order to make our model less complicated without losing the essential physics, we make the approximation of neglecting these inter-dot contributions. We also assume that tunneling energies for different orbitals in Eq. 3.12 are equal, \( g_{\alpha i,\beta j} \equiv g \).

Noting that the ground state of two electrons in a quantum dot at low magnetic fields is a singlet and the next three higher energy states are triplets and denoting the energies of the singlet (\( S \)) and triplet (\( T \)) states of the left (L) and right (R) quantum dot by \( E_{S/R}^{L/R} \), we can then write the forms of \( J_i \) and \( J'_{ij} \) in terms of the tunneling energies and singlet and triplet energies. They are given by

\[
J_1 \approx \frac{2g^2}{E_R^S - E_L^S}, \quad J_2 \approx \frac{2g^2}{E_R^T - E_L^T},
\]

\[
J'_{12} = J'_{21} = \frac{J_1 + J_2}{2} \equiv J'.
\]

The couplings due to these exchange terms are shown schematically in Fig. 3.4. Here, we assume that \( E_R^S - E_L^T > 0 \) is satisfied due to detuning, tighter right quantum dot confinement or some experimental scheme that is employed. Then, \( E_R^T - E_L^S > 0 \) follows because the triplet states are higher in energy than the singlet state, \( E_L^T > E_L^S \). Note that in truncating the series in Eq. 3.13, we also implicitly assumed that \( g < (E_R^S - E_L^T) \).

For the form of intra-dot energy operator given in Eq. 3.10, the singlet and triplet
energies are
\[
E^L_S = 2e_{L1} + C_{L1,L1} + 2\mu_L,
E^R_S = 2e_{R1} + C_{R1,R1} + 2\mu_R,
E^L_T = e_{L1} + e_{L2} + C_{L1,L2} - K_{L1,L2} + 2\mu_L,
E^R_T = e_{R1} + e_{R2} + C_{R1,R2} - K_{R1,R2} + 2\mu_R.
\]

(3.22)

The logical basis given in Chapter 3 is in spin form. To make it consistent with the
notations introduced here, we explicitly write them out below. Here, the three-electron kets
are the usual Slater determinants.
\[
|0\rangle_L \equiv |S\rangle |\downarrow\rangle = |\phi_{L1\uparrow}(r_1) \phi_{L1\downarrow}(r_2) \phi_{R1\downarrow}(r_3)\rangle,
|1\rangle_L \equiv \sqrt{\frac{1}{3}}|T_0\rangle |\downarrow\rangle - \sqrt{\frac{2}{3}}|T_-\rangle |\uparrow\rangle
= \sqrt{\frac{1}{6}} \left[ |\phi_{L1\uparrow}(r_1) \phi_{L2\downarrow}(r_2) \phi_{R1\downarrow}(r_3)\rangle + |\phi_{L1\downarrow}(r_1) \phi_{L2\uparrow}(r_2) \phi_{R1\downarrow}(r_3)\rangle \right]
- \sqrt{\frac{2}{3}} \left[ |\phi_{L1\downarrow}(r_1) \phi_{L2\downarrow}(r_2) \phi_{R1\uparrow}(r_3)\rangle \right].
\]

(3.24)

The effective Hamiltonian in the logical basis \{\(|0\rangle_L, |1\rangle_L\}\} is given by
\[
\hat{H}^{(t-J)}_{\text{eff}} = \begin{pmatrix}
-J_1 & \sqrt{\frac{3}{2}}J' \\
\sqrt{\frac{3}{2}}J' & E_{ST} - \frac{3}{2}(J_1 + J_2)
\end{pmatrix},
\]

(3.25)

with \(E_{ST} \equiv E^T_{ST} - E^L_{ST}\) being the singlet-triplet splitting of the two electrons in the left quan-
tum dot. Of significance is the exchange term \(J'\) whose expression is given in Eq. (3.21) which
couples the two logical states of the qubit and can be modulated by means of electrically
controlling the tunnel barrier between the quantum dots.

3.8 Connectivity between two hybrid qubits

This section discusses why the connectivity of two hybrid qubits composed of six electrons
in four dots arranged in a linear array (Fig. 3.1(d) and Fig. 3.1(f) of Chapter 3) is higher
than the connectivity of two qubits that consist of electrons in six dots arranged in a linear array (shown in Fig. 3.1(e)). We have calculated the number of independently controllable interactions using the methods described in the previous section and find that there are two independent interdot couplings when one dot has two electrons and the other has one, and that there are three independent interdot couplings when both dots have two electrons. The coupling strengths are tunable as a function of the top-gate voltages that define the quantum dots. As discussed in the previous section, the control parameters include the height of the tunnel barrier, the energy splitting between the ground and excited states in each dot, and the energy detuning between the two dots. Here we present intuitive arguments that the electrons in the dots have at least two independently tunable interdot couplings for both cases. Note that our searches for CNOT gate sequences, described in the following section, require only two independent interdot couplings, for either of the connectivities shown in Figs. 3.1(d) and (f).

We first consider the case of Fig. 3.1(d), where one dot has two electrons and the other dot has one electron. Here, since the geometry of the connections between hybrid qubits is the same as the connections between the dots comprising a single logical qubit, the analysis presented in the previous section can be applied straightforwardly. Fig. 3.5 is a visualization of configurations of the two dots that have different effective exchange couplings, because they involve overlaps of different orbital states of the doubly occupied dot. (One can view the diagram as indicating the energies and extents of single-particle orbitals for the possible electron states of the dot.) Note that configuration (3) in Fig. 3.5(a) does not occur during single-qubit manipulation, but it becomes accessible when two qubits are coupled. Fig. 3.5(b) shows a similar schematic when the connection between qubits is between two dots, each with two electrons. Because the two dots are not identical, configurations (1) and (2) are independently tunable. Configuration (3) contributes because of tunneling to excited states. Configuration (4) is a configuration that cannot occur for a single qubit, but which can appear during two-qubit manipulation. Because the number of configurations with different exchange couplings is larger than for the case shown in (a), it is extremely
plausible that the number of independently tunable interactions is at least as great. This expectation is borne out by
detailed calculations using methods similar to those used in the previous section.

Figure 3.5: (a) Schematic diagram of the (2,1) electron configurations of one doubly oc-
cupied dot and one singly occupied dot that have separately tunable tunnel couplings. Con-
figuration 3, on the right, does not arise when manipulating a single qubit, but can
arise during two-qubit manipulations. Using the methods presented in Section 3.7 we have
performed calculations with properly symmetrized wavefunctions, showing that the (2,1)
configuration has two independently tunable exchange couplings. (b) Schematic diagram
of the (2,2) electron configurations of two doubly occupied dots that have separately tun-
able tunnel couplings. Configuration 4, on the right, does not arise when manipulating a
single qubit, but can arise during two-qubit manipulations. The number of distinct tunnel
couplings is greater than for the (2,1) case shown in (a), so it is plausible that the number
of tunable interdot exchange couplings is at least as great as in (a). Detailed calculations
show that the (2,2) arrangement has three independently tunable exchange couplings. Our
calculations of gate sequences presented in the next section assume only two independently
tunable interdot exchange couplings.

plausible that the number of independently tunable interactions is at least as great. This
expectation is borne out by detailed calculations using methods similar to those used in the
previous section.

3.9 CNOT gates for qubits with increased connectivities

This section presents gate sequences that implement gates that are equivalent to CNOT, up
to local unitary transformations, for two logical qubits that consist of the two eigenstates
of three spins with $S = 1/2, S_z = -1/2$. The spin symmetries of the hybrid qubit proposed
here is the same as the triple-dot qubit of Ref. [58], however, two hybrid qubits consisting
of six electrons in four dots arranged in a linear array has more spin-spin couplings than
two logical qubits consisting of six electrons in six dots arranged in a linear array, as shown in Fig. 3.1. Here we present evidence that this increased connectivity can be exploited to reduce the number of gate operations needed to implement gates of two logical qubits, by presenting two-spin gate sequences that are equivalent to a CNOT gate, up to local unitary operations, for different connectivities of the spin-1/2 particles that make up the logical qubits. This result is not unexpected, since Ref. [58] shows that for single qubit operations, at most three gates are needed when all three spins are coupled (e.g. in a triangular quantum dot configuration), and at most four gates are necessary when the spins are coupled only to nearest neighbors in a linear array.

The quantum dot geometries we considered are shown in panel (a) of Figs. 3.6 and 3.7. We implemented a search algorithm similar to the one described in Ref. [59], which is a combination of Nelder-Mead [84] and genetic [85] algorithms. We parallelized the program and exploited the resources of the University of Wisconsin Center for High Throughput Computing (CHTC) [86] grid using Condor [87] to improve the speed of the calculations.
As in Ref. [59], the numerical results could be used to deduce exact gate times, which points to the power of the algorithm and perhaps some underlying hidden symmetry of the $S = 1, S_z = 1$ subspace in which the two logical qubits reside. The exact gate sequences are presented schematically in Fig. 3.6(b) and Fig. 3.7(b).

We also obtained the local unitary operations necessary for obtaining the exact CNOT gate for the geometry of Fig. 3.7(a), and the exact times and spin couplings are presented in Fig. 3.7(b). Note that in this coded qubit scheme, the local unitary operations are implemented as exchange gates.
3.10 Details of the experimental measurements

This section presents the details of the experimental measurements shown in Fig. 3.2. The data in Fig. 3.2 correspond to a single electron tunneling between the left dot and the left reservoir, as shown in the insets to Fig. 3.2(b–d). Charge sensing is performed by measuring the current $I_{QPC}$ using a bias voltage of 500 µV. We measure the electron loading and unloading rates by applying a small amplitude square wave pulse to one of the gates defining the quantum dot and observing the electron tunneling events, which are manifested as steps in $I_{QPC}$. We report the amplitude of the square wave pulse as generated by the arbitrary waveform generator (AWG); there is attenuation between the AWG and the sample. To measure the tunnel rate into the dot, we start with one electron in the left dot and apply a step increase in voltage of 80 mV to the experimental wiring leading to gate PL, causing an electron to tunnel from the lead into the dot. By averaging 300 single-shot traces of $I_{QPC}$ versus time, we obtain the curves shown in Fig. 3.2(b–c), which show an exponential decay (increase) for electron loading (unloading) events. By fitting these curves to exponentials, we extract the tunnel rates for electron loading and unloading. To study the singlet and triplet states independently, we apply an in-plane magnetic field $B$. At small $B$, the singlet state $S$ is the ground state, whereas at large $B$ the triplet state $T_-$ is the ground state. Applying a small amplitude voltage pulse will enable loading of the ground state only. Thus, a measurement at both small and large $B$ of the tunnel rate into or out of the dot with a small amplitude voltage pulse enables a measurement of the tunnel rates into and out of both the singlet and the triplet states. As discussed in Chapter 2, we find these tunnel rates to be quite different, enabling readout of the spin state.

The difference in the unloading rate of the singlet $S$ and the triplet $T_-$ can be used to measure the triplet-singlet relaxation time at magnetic fields $B$ small enough that the singlet $S$ is the ground state [73]. We enable loading of both state $S$ and state $T_-$ by applying a voltage pulse to the dot, as described in Section 3.5. We choose a pulse amplitude that brings $T_-$ into resonance with the Fermi level of the lead, preferentially loading that state. With this initial condition, we then measure the unloading rate of the dot as a function of
Wave generator and applied to the experimental wiring leading to gate PL. The loading and unloading voltage levels correspond to voltages generated by the arbitrary 120 mV, as in panel (c), the unloading rate is fast, indicating that the state $T^-$ is long. This behavior indicates that during a long loading period the electron loads into state $T^-$ and $T^0$ relax to the singlet state $S$ before the electron tunnels out of the dot. The loading and unloading voltage levels correspond to voltages generated by the arbitrary wave generator and applied to the experimental wiring leading to gate PL.

The dwell time $\Delta t$ between the steps in voltage that induce loading and unloading of an electron. The unloading rate shows an exponential decay as a function of dwell time, as reported in Fig. 3.2(d). The unloading rate is fast when the electron stays in the $T^-$ state throughout the dwell time, so that the electron tunnels out of the dot from the $T^-$ state; the unloading rate is slow when the electron decays to the singlet state before exiting the dot. A measurement of the characteristic time of this decay is thus a measurement of $T_1$ for the triplet-to-singlet relaxation process.

To perform this measurement, it is useful to find the loading voltage that corresponds to resonance with the $T^-$ state, by measuring the loading and unloading tunnel rates as a function of the amplitude of the loading pulse. As shown in Fig. 3.8, the loading rate develops two peaks as we increase the amplitude of this pulse. We identify the two peaks as $T^-$ and $T^0$ states coming into resonance with the Fermi level. These states have fast tunnel rates because the triplet wavefunction is more strongly coupled to the lead. The splitting between the peaks, when converted to energy using a lever arm measured in [16] ($\alpha = 0.00144 \pm 0.00013 \text{eV/V}$), is $144 \pm 13 \text{eV}$ between triplet $T^0$ and $T^-$, a value that is in
good agreement with the Zeeman splitting at $B = 1.5 \, \text{T}$, $\Delta E_B = 138 \, \mu\text{eV}$, where we have used the $g$-factor measured in [16]: $g = 1.585$ at $B = 1.5 \, \text{T}$. The unloading rate, when the pulse amplitude is such that the $T_-$ state is in resonance with the Fermi level, is fast when the loading time is short, and slow when the loading time is long. This behavior indicates that during a long loading period the electron loads into state $T_-$ and then relaxes to state $S$. As shown in Fig. 3.8(c), for short loading times the electron remains in state $T_-$ until it unloads.

### 3.11 Summary

To summarize, we have proposed a solid state qubit architecture which we term hybrid qubit. It consists of three electrons in two quantum dots. Compared to previous proposals, this new qubit has the important advantages of fast gate operations and relative simplicity of fabrication. Experimental data are presented that support the feasibility of constructing the qubit architecture using Si/SiGe quantum dots.
Chapter 4

Coherent quantum oscillations and echo measurements of a Si charge qubit

4.1 Introduction

Fast, coherent control of charge qubits has been demonstrated in both superconducting circuits [88, 89] and III-V semiconductor quantum dots [80, 90]. Beyond its intrinsic interest, understanding semiconductor charge qubit coherence is also important for spin qubits: mixing spin with charge degrees of freedom (either through spin-orbit coupling [91] or the exchange interaction [3, 60, 92, 93, 12, 94]) enables faster spin manipulation than would otherwise be possible. When this mechanism is used, charge coherence can determine the ultimate fidelity of a spin qubit [94].

In this chapter, we present experimental measurements of fast coherent quantum oscillations between the (2,1) and (1,2) charge states of a qubit formed in a Si/SiGe double quantum dot.

Given a Bloch sphere [95] with the ±z axes representing the (1,2) and (2,1) states, rota-

\footnote{This chapter is adapted from Ref. [13].}
tions about the $x$-axis, or Larmor oscillations, are observed when the (2,1) and (1,2) states are energetically degenerate, with a decoherence time $T_2^* = 2.1$ ns. Rotations about the $z$-axis are probed in a Ramsey fringe experiment and are observed with a shorter coherence time, $T_2^* = 127$ ps. The Ramsey fringes are measured when the charge qubit is operated in a regime where the energy difference between the qubit states depends strongly on detuning $\varepsilon$, the energy difference between the (2,1) and (1,2) charge states, and so is highly sensitive to charge noise in the local environment. The significantly different coherence times are consistent with the dominant dephasing mechanism arising from fluctuations in $\varepsilon$, as has previously been observed in GaAs devices [90]. We also present measurements of charge echo in a semiconductor quantum dot charge qubit, with the echo sequence yielding an increase of the coherence time from 127 ps to 760 ps. The results demonstrate control of a silicon charge qubit, and they show that charge echo can be exploited to improve its coherence.

### 4.2 Larmor oscillations (x-rotations)

The device measured in the experiment was fabricated in a Si/SiGe heterostructure as described in Refs. [96, 116]; a scanning electron microscope image of an identical device is shown in Fig. 4.1(a). The quantum point contact on the right side of the double dot is used as a charge sensor. Using magnetospectroscopy measurements [11], we confirm the valence charge occupation of the double dot is (2,1) or (1,2); either or both dots may contain a closed shell beneath the valence electrons, although if present such shells do not appear to play a role in the work we report.

Fig. 4.1(b) shows an energy level diagram of the anticrossing between (2,1) and (1,2) as a function of detuning energy $\varepsilon$. Near the charge degeneracy point ($\varepsilon = 0$) the system is well-described by the Hamiltonian of a two-state system:

$$H = \begin{pmatrix} \varepsilon/2 & \Delta \\ \Delta & -\varepsilon/2 \end{pmatrix}.$$  \hspace{1cm} (4.1)
Figure 4.1: (a) SEM image of a device identical to the one used in the experiment. The current $I_{\text{QPC}}$ is used for charge sensing through a measurement of its transconductance $G_L = \partial I_{\text{QPC}}/\partial V_L$, and voltage pulses are applied to gate L. (b) Bottom: Diagram of energy levels versus detuning $\varepsilon$, showing the anticrossing between (2,1) and (1,2) charge configurations. The red arrow represents an applied voltage pulse with the dc level and amplitude of the pulse chosen to place the peak of the pulse at $\varepsilon = 0$. Top: Example pulse showing the pulse peak position $\varepsilon_p$, pulse duration $t_p$, and the measurement phase. (c) Larmor oscillations between the (2,1) and (1,2) charge configurations, as a function of pulse duration $t_p$ and pulse peak position $\varepsilon_p$. The Larmor oscillations that reflect rotations between the states with (2,1) and (1,2) charge occupations are manifest near $\varepsilon_p = 0$. (d) Numerical simulation of the Larmor oscillations of (c), using an 80 ps pulse rise time and the energy level diagram in (b) with best fit parameter $\Delta = 10.8 \mu$eV ($\Delta/h = 2.62$ GHz). (e) Solid red: a horizontal cut taken near zero detuning from the integrated data of (c), showing Larmor oscillations as a function of pulse duration. Dashed black: a corresponding cut from the simulated charge occupation data.

Coherent oscillations between the two charge states can be observed when the detuning $\varepsilon$ is changed abruptly. For example, starting in a position eigenstate with charge occupation (2,1) at large negative $\varepsilon$, after increasing the detuning suddenly to $\varepsilon = 0$, as shown by the red horizontal arrow in Fig. 4.1(b), the system Hamiltonian becomes $H = \Delta \sigma_x$, where $\sigma_x$ is the usual Pauli matrix. Subsequently, the system oscillates between (2,1) and (1,2) at the Larmor angular frequency $2\Delta/h$. More generally, non-adiabatically increasing the detuning $\varepsilon$ to a value $\varepsilon'$ is expected to induce oscillations at the angular frequency $\Omega_R = \sqrt{\varepsilon'^2 + 4\Delta^2}/h$ about a tilted axis: as one moves away from the polarization line at which $\varepsilon = 0$, the oscillations increase in frequency and decrease in amplitude.
Figure 4.1(c) shows a number of Larmor oscillations between the (2,1) and (1,2) charge states. Square pulses of duration $t_p$ and amplitude $V_p = 800$ mV are applied to gate L at frequency 40 MHz. The transconductance $G_L = \partial I_{QPC} / \partial V_L$ is plotted as a function of $t_p$ and $\varepsilon_p$, the position in detuning of the peak of the pulse. The connection between $V_L$, $t_p$, $\varepsilon_p$, and other details can be found in Section 4.5. Oscillations of the signal are apparent out to more than three nanoseconds.

To enable comparison to theory, and to obtain quantitative dephasing times from the experiment, we integrate the data presented in Fig. 4.1(c) and extract the probability $P_{(1,2)}$ of occupying the (1,2) charge state (see Section 4.5 for details). Fig. 4.1(e) presents the charge oscillation near zero detuning from the integrated data of (c) as the solid red trace. By fitting the amplitude of the oscillations to exponential decays, we extract a dephasing time $T_2^* = 2.1 \pm 0.4$ ns near $\varepsilon_p = 0$, marked by label B in Fig. 4.1(b). This dephasing time is relatively long, because at the anticrossing the difference in energy between the eigenstates is insensitive to detuning fluctuations.

Numerical simulations of the experiment were performed based on the energy level diagram in Fig. 4.1(b). We model the dynamical evolution of the density matrix $\rho$ of the system as a function of detuning $\varepsilon_p$ and pulse duration $t_p$ using a master equation [95, 97]:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + D,$$

(4.2)

where $D$ is a phenomenological term that describes pure dephasing of the charge state under the assumption of Markovian dynamics. $D$ is given, in the $\{|(2,1), (1,2)\}$ basis, by

$$D = \begin{pmatrix} 0 & \Gamma_0 \rho_{12} \\ \Gamma_0 \rho_{21} & 0 \end{pmatrix},$$

(4.3)

where $\Gamma_0 = 0.48$ GHz is the dephasing rate ($1/T_2^*$) measured near zero detuning. This dephasing can have contributions both from the effect of charge noise on the tunnel coupling and from second-order effects from detuning noise, since the qubit is to first order insensitive to detuning noise at the charge degeneracy point. The (2,1) and (1,2) occupation probabilities are extracted at the end of the $t_p$-pulse and, for the duration of the measurement
phase, are allowed to relax exponentially to the ground state \((2,1)\) with a relaxation time \(T_1\). The simulated charge occupation is determined by averaging the charge state for the entire 25 ns pulse period. Low-frequency fluctuations in the detuning \(\varepsilon\) are incorporated following Ref. [90], by performing a convolution of the results at each \(\varepsilon\) with a Gaussian in \(\varepsilon\) of width \(\sigma_\varepsilon = 5 \text{ } \mu\text{eV}\). The best fit to the data is found with a charge relaxation time \(T_1 = 18 \text{ ns}\). The results obtained using the tunneling amplitude \(\Delta = 10.8 \text{ } \mu\text{eV} \ (\Delta/h = 2.62 \text{ GHz})\), shown in Fig. 4.1(d), agree well with the data. Fig. 4.1(e) is a horizontal cut showing the measured and simulated charge occupation data near \(\varepsilon = 0\). Again, good agreement between the data and the calculation is found.

### 4.3 Ramsey fringes (z-rotations)

We now demonstrate coherent rotations of the charge qubit about the \(z\)-axis of the Bloch sphere by performing a Ramsey fringe experiment [98, 99], using the two-pulse sequence shown in the inset to Fig. 4.2(a). Starting at a negative detuning in the \((2,1)\) state, the qubit is pulsed to the \((2,1)-(1,2)\) anticrossing, which causes the Bloch vector to rotate around the \(x\)-axis. The duration of this first pulse is chosen (based on the data in Fig. 4.1(c)) so that the Bloch vector is rotated around the \(x\)-axis by a nominal angle of \(5\pi/2\), taking it from being along \(z\) to being in the \(x\)-\(y\) plane (we use a \(5\pi/2\) pulse of amplitude 600 mV and duration 280 ps instead of a \(\pi/2\) pulse, because of the difficulty of applying high-quality pulses shorter than 100 ps). After a variable free evolution time \(\tau\) at the base level of the detuning \(\varepsilon_b\), during which the Bloch vector rotates about the \(z\)-axis, a second pulse is applied to rotate the state about the \(x\)-axis on the Bloch sphere by another \(5\pi/2\). The charge measured at the end of this process oscillates as a function of the time \(\tau\) between the two pulses at a frequency determined by the difference in energy of the states involved at the base level of detuning.

Fig. 4.2(a) shows the transconductance \(G_L\) of the charge sensor as a function of the base level detuning \(\varepsilon_b\) and the time \(\tau\), in the presence of the two-pulse pattern applied at a repetition rate of 25 MHz. For very short \(\tau\), the \(5\pi/2\) pulses overlap and one is essentially
Figure 4.2:  (a) Ramsey fringes: QPC transconductance $G_L$ as a function of the base level of detuning $\varepsilon_b$ and the time $\tau$ between two $5\pi/2$ pulses, as shown in the inset. The oscillations observed in the region where $\tau > 280$ ps reflect the rotation of the Bloch vector around the $z$ axis in the $x-y$ plane. (b) Gray: a line cut of the integrated data of (a) after removal of a smooth background, as described in Section 4.5. Red: fit to the form $A \exp\left[-\left(\tau - \tau_0\right)^2 / T_2^*\right] \cos(\omega t + \phi) + C$, which yields $T_2^* = 127 \pm 8$ ps. (c) Charge echo: QPC transconductance $G_L$ as a function of $\varepsilon_b$ and $\delta t$ for $t = 640$ ps. Oscillations are strongest near $\delta t = 0$, where equal free evolution times before and after the $3\pi$ $x$-rotation provide the best correction for slow inhomogeneous dephasing. Away from $\delta t = 0$, increasingly mismatched free evolution times provide less correction, and the oscillations decay with characteristic time $T_2^*$. Inset: Trace of the pulse sequence used to acquire these data. (d) Sequence of pulses used to implement charge-echo: a nominal $5\pi/2$ $x$-rotation into the $x-y$ plane, free evolution for a time $t/2 + \delta t$, a $3\pi$ $x$-rotation, free evolution time $t/2 - \delta t$, and a $5\pi/2$ $x$-rotation out of the $x-y$ plane. For this experiment, $5\pi/2$ pulses have a duration of 280 ps and $3\pi$ pulses have a duration of 330 ps. (e) Dark circles: charge oscillation amplitudes $\Delta P_{(1,2)}$ as a function of the free evolution time $t$, extracted from data sets with different $t$ values. The echo amplitude decays as $t$ increases. A fit of the decay to the Gaussian form $y_0 + A \exp\left[-\left(t/T_2\right)^2\right]$ yields $T_2 = 760 \pm 190$ ps, significantly longer than $T_2^*$. Thus, the echo pulse sequence corrects for slow inhomogeneous dephasing and extends the coherence time.

performing a Larmor oscillation experiment. At $\tau \approx 280$ ps, the time interval between the end of the first $5\pi/2$ pulse and the start of the second becomes nonzero, and the observed oscillations correspond to a Ramsey fringe measurement.

To analyze these data quantitatively, we again integrate the data and extract $P_{(1,2)}$. Fig. 4.2(b) shows a cut through the integrated data at the value of detuning $\varepsilon_b = -120 \mu eV$ marked by the arrow labeled A in Fig. 4.1(b), after subtraction of a smooth background
These Ramsey fringes oscillate at 28 GHz, which agrees with the energy difference of the two charge states at that detuning value. We fit these oscillations to the product of a cosine function and a Gaussian. This procedure yields $T_2^* = 127 \pm 8$ ps, much shorter than $T_2^* = 2.1$ ns measured in the Larmor experiment. As is clear from Fig. 4.1(b), at the large negative detuning where the oscillations are being generated, the energy levels diverge rapidly from each other as a function of $\varepsilon$, providing no protection from charge noise. The dephasing time in this Ramsey fringe experiment is nonetheless twice as long as the value of 60 ps obtained by measuring Ramsey fringes for a GaAs charge qubit and using a similar fitting procedure to extract $T_2^*$. Calculations following the methods of Ref. 79 show that the charge dephasing rate in GaAs from polar optical phonons may be of order 1 GHz, whereas similar calculations for phonon-induced charge dephasing in Si yield values of order 0.5 MHz. Thus, in both materials, and particularly in Si, improvements may be possible through a reduction of excess charge noise.

### 4.4 Charge-echo experiment

We now demonstrate that the effects of inhomogeneous dephasing can be ameliorated using a charge-echo method. Charge-echo is implemented by applying the voltage pulse sequence shown in Fig. 4.2(d). When the tips of the pulses reach the (2,1)-(1,2) anticrossing, the pulse sequence consists of a $5\pi/2$ pulse (which rotates the Bloch vector into the $x$-$y$ plane), a free evolution at the base detuning for a time $t/2 + \delta t$, a $3\pi$ pulse (which flips the Bloch vector to its mirror image with respect to $x$-$z$ plane), a second free evolution at the base detuning for a time $t/2 - \delta t$, and a second $5\pi/2$ pulse (which rotates the Bloch vector about the $x$-axis again). Fig. 4.2(c) shows the transconductance measured as a function of the detuning $\varepsilon_b$ and the time $\delta t$ for $t = 640$ ps. The oscillations are strongest around $\delta t = 0$, where the echo sequence best corrects for inhomogeneous dephasing. As $|\delta t|$ increases, more time is spent performing an uncorrected $z$-rotation, and the oscillation amplitude decays with characteristic time $T_2^*$, just as in the Ramsey fringe experiment. As the total free evolution time $t$ increases, the oscillation amplitude will
decay with characteristic time $T_2$. To extract $T_2$, we perform the echo pulse sequence for multiple values of $t$. We convert the the transconductance data to charge occupation data and extract the amplitude of the charge oscillation $\Delta P_{(1,2)}$ (see Section 4.6). Fig. 4.2(e) shows the extracted value of $\Delta P_{(1,2)}$ for each data set, plotted as a function of $t$. The echo amplitude clearly decays as $t$ is made longer. A Gaussian fit of the decay yields $T_2 = 760 \pm 190$ ps. The significant increase in coherence time indicates that low-frequency noise processes play an important role in limiting qubit coherence.

4.5 Measurement details and line cuts of Larmor and Ramsey oscillations

The data shown in Fig. 4.1(c) of the is acquired by sweeping the voltage $V_L$ on gate L as a function of the pulse duration $t_p$. As $t_p$ increases, because of the high-pass filter in the bias-tee for the high frequency line, the time-averaged voltage on gate L changes, resulting in a linear change in the relationship between $V_L$ and detuning $\epsilon$. Thus, to remove this offset, before converting to $\epsilon$ we shift each vertical scanline by an amount $\delta V_L = V_p \times t_p \times f_{rep}$, where $V_p$ is the pulse amplitude and $f_{rep}$ is the pulse repetition frequency. The vertical axis in Fig. 1(c) then is converted to $\epsilon_p$, the detuning value at the peak of the pulse, by noting that the main, slowest oscillation corresponds to $\epsilon_p = 0$, and by fitting the change in Larmor frequency as a function of $V_L$ for small positive $\epsilon_p$. For our experimental conditions, one Volt applied to the high frequency coaxial line connected to gate L results in a change in gate voltage of 6.6 mV on that gate. The connection between $V_L$ and detuning $\epsilon$ is $\alpha_{L,\epsilon} = -24 \mu eV/mV$ for the Larmor oscillation data and $\alpha_{L,\epsilon} = -36 \mu eV/mV$ for the Ramsey fringe and charge echo data (these data sets were acquired at different gate voltage tunings of the dots). The lever arm is determined by fitting the Larmor oscillation frequency as a function of gate voltage $f = \sqrt{\alpha_{L,\epsilon} V_L^2 + 4 \Delta^2 / h}$.

Figure 4.3(a) shows the results of the integration of the data in Fig. 4.2(a), normalized
Figure 4.3: Ramsey fringe analysis. (a) Integration of the transconductance $G_L$ from Fig. 4.2(a) yields the probability $P_{(1,2)}$ of occupying the (1,2) charge state in the regime where the (2,1) charge state is the ground state. The data is normalized by noting that the total charge transferred across the polarization line is one electron. The red dashed box indicates the location of the fringes. (b) Dark gray: line cut of the data in (a), at the detuning $\varepsilon = -120 \mu eV$. Light gray: Smooth background that is subtracted from the line cut before fitting the data to a damped sinusoidal form. (c) (The same as Fig. 4.2(a)) Gray: the data from (b) after subtraction of the smooth background. Red: fit to the form $A \exp\left(-\frac{(\tau - \tau_0)^2}{T_2^*}\right) \cos(\omega t + \phi) + C$, which yields $T_2^* = 127 \pm 8$ ps.

to obtain the probability $P_{(1,2)}$ of being in the (1,2) charge configuration when (2,1) is the ground state. Fig. 4.3(b) shows a line cut through the plot in Fig. 4.3(a) at the value of $\varepsilon = -120 \mu eV$. Fig. 4.3(c), which is the same as Fig. 4.2(c), shows the Ramsey fringes after subtraction of the smooth background shown in Fig. 4.3(b).

4.6 Analysis of charge echo experiment

To extract $T_2$ from the charge echo data we perform two analyses. In the first, the oscillation amplitude is quantified at a given value of the detuning by analyzing the fast Fourier transform (FFT) of the probability $P_{(1,2)}$. In the second, the oscillation amplitude is quan-
tified at a given oscillation frequency by analyzing the FFT of $G_L$. As shown below, the results from the two methods are consistent.

The results presented in Fig. 4.2 of the main text were obtained by analyzing the oscillation amplitude of $P_{(1,2)}$ at fixed detuning. To get $P_{(1,2)}$ as a function of detuning and $\delta t$, we integrate the time-domain data (such as that shown in Fig. 4.4(a–c)) from top to bottom. After removing a linear background, we normalize by noting that the total charge transferred across the polarization line is one electron. We perform an FFT (using Igor Pro [101]), and to ensure that the FFT magnitude is comparable for different values of $t$, we use the same number of points (or equivalently, the same length of time) from each data set by taking a 364 ps cut centered about $\delta t \approx 0$. The transforms are shown in Fig. 4.4(d–f). The oscillations of interest appear as spectral weight that moves to higher frequency at more negative detuning (farther from the anti-crossing). Moreover, for each value of the detuning, the FFT magnitude is nonzero over a certain range of frequencies.

To extract the charge oscillation amplitude from these FFTs, we first take a horizontal trace from the FFT data at $\varepsilon = -120 \mu$eV, where the pulse tip is around zero detuning. We then integrate the trace over a bandwidth region from 46 to 72 GHz. Because for the shortest free evolution time $t$ the oscillations in $P_{(1,2)}$ can be extracted easily from the untransformed data, we use that oscillation amplitude to normalize each of the FFT integrations, allowing us to plot a normalized oscillation amplitude as a function of $t$ in Fig. 4.4(g). The echo amplitude decays as the free evolution time $t$ is made longer, with a characteristic time $T_2$. Fitting a Gaussian to the decay yields $T_2 = 0.76 \pm 0.19$ ns.

For comparison, we also extract $T_2$ from an analysis of the FFT of the unintegrated transconductance data at fixed oscillation frequency. We perform an FFT with a square window function on a 386 ps cut centered at $\delta t \approx 0$ and plot the magnitude as a function of detuning and frequency, as shown in Fig. 4.4(h–j). We take the magnitude at the point where the central feature (black line) intersects 65 GHz and plot this quantity as a function of $t$ (Fig. 4.4(k)). Fitting to a Gaussian decay yields $T_2 = 0.62 \pm 0.14$ ns. As Fig. 4.4 demonstrates, the two approaches of extracting $T_2$ yield similar results.
Figure 4.4: Analysis of echo data for extraction of the decoherence time $T_2$. (a–c) Transconductance $G_L$ as a function of the base level of detuning $\varepsilon$ and $\delta t$ (defined in Section 4.4) for total free evolution times of $t = 390$ ps, $t = 690$ ps, and $t = 990$ ps, respectively. (d–f) Fourier transforms of the charge occupation $P_{(1,2)}$ as a function of detuning $\varepsilon$ and oscillation frequency $f$ for the data in (a–c), respectively. We obtain $P_{(1,2)}$ (not shown here) by integrating the transconductance data in (a–c) and normalizing by noting that the total charge transferred across the polarization line is one electron. Fast Fourier transforming the time-domain data of $P_{(1,2)}$ allows us to quantify the amplitude of the oscillations visible near $\delta t = 0$. The oscillations of interest appear as weight in the FFT that moves to higher frequency at more negative detuning (farther from the anti-crossing). For an individual detuning energy, the FFT has nonzero weight for a nonzero bandwidth. (g) Echo amplitude as a function of free evolution time $t$. The data points (dark circles) are obtained at $\varepsilon = -120 \, \mu eV$ by integrating a horizontal line cut of the FFT data over a bandwidth range of 46-72 GHz, then normalizing by the echo oscillation amplitude of the first data point, as described in the text. The echo oscillation amplitudes, plotted for multiple free evolution times, decay with characteristic time $T_2$ as the free evolution time $t$ is made longer. By fitting the decay to a Gaussian, we obtain $T_2 = 760 \pm 190$ ps. (h–j) Fourier transforms of the transconductance $G_L$ as a function of $\varepsilon$ and oscillation frequency $f$ for (a–c), respectively. As $t$ is increased, the magnitude for oscillations at a given frequency decays with characteristic time $T_2$. We take the magnitude of the FFT at the point where the central feature (black line) intersects 65 GHz. (k) Measured FFT magnitudes at 65 GHz for multiple free evolution times (dark circles) with a Gaussian fit (red line), which yields $T_2 = 620 \pm 140$ ps, in reasonable agreement with the result shown in (g).
4.7 Summary

In summary, we have observed coherent quantum charge oscillations in a qubit formed in a Si/SiGe double quantum dot. The coherence time $T^*_2$ is 2.1 ns for Larmor oscillations or x-rotations, at the charge degeneracy point, and is 127 ps for Ramsey fringes that reflect rotations about the z-axis of the Bloch sphere. Implementation of a charge-echo pulse sequence increases the decoherence time from 127 ps to 760 ps in the regime where the energy difference between the two qubit states depends substantially on detuning.
Chapter 5

Fast coherent manipulation of three-electron states in a double quantum dot\(^1\)

5.1 Introduction

Electrons in semiconductor quantum dots are promising candidates for use in quantum computing, because of the potential of this platform to enable coherent quantum control on large numbers of qubits [3]. Much recent progress has led to demonstrations of both spin- and charge-based qubits in both GaAs and Si [80, 29, 48, 102, 18, 103, 90, 99, 13, 104, 105]. Charge qubits can be manipulated quickly but have relatively short coherence times, while spin qubits have long coherence times but long manipulation times. The tendency of the speed of manipulation to be correlated with the rate of decoherence is not surprising, because both depend on the coupling of the qubit to external degrees of freedom (designed and wanted for manipulation, and extraneous and unwanted for decoherence). Refs. [12, 94], describing a quantum dot hybrid qubit, present theoretical arguments that a system with more degrees of freedom, specifically three electrons in two quantum dots, can overcome

\(^1\)This chapter is adapted from Ref. 14.
this tendency. Two of the states can form a qubit with spin character that has a long coherence time. By accessing a third state via a charge transition, fast operations can be performed, and then the qubit can be converted back into a spin-like qubit with long coherence time. Such a strategy requires that one can systematically and accurately control transitions between several different quantum states of the same system.

This chapter presents experiments that demonstrate the ability to tailor transitions between quantum states of three electrons in two quantum dots. The experiments are performed on a double quantum dot fabricated in a Si/SiGe heterostructure, as described in Ref. [38, 16, 13]; a scanning electron microscope image of an identical device is shown in Fig. 5.1(a). By adjusting the gate voltages appropriately, we tune the dot occupation so that the valence charge occupation of the double dot is (2,1) or (1,2), where the first (second) number is the charge occupation in the left (right) dot, as confirmed by magnetospectroscopy measurements [11].

In this three-electron double dot system, four states are important, the ground \(|0\rangle\) and first excited state \(|1\rangle\) of the dot in the (2,1) charge occupation, and the corresponding ground \(|2\rangle\) and first excited state \(|3\rangle\) of the dot in the (1,2) charge occupation. The qubits are manipulated by pulsing a voltage that changes the detuning \(\varepsilon\), which is the energy difference between the two dots. By applying appropriate sequences of voltage pulses, oscillations between different pairs of quantum levels can be induced. Because oscillations with periods much shorter than the rise times of the applied pulses can be excited, and because quantum oscillations between some pairs of the states are quite insensitive to the dominant dephasing mechanism, which is fluctuations in the value of the detuning \([106]\), many (over a hundred) oscillations can be observed within one coherence time. The consistency of our interpretation of the data in terms of coherent quantum oscillations between different energy levels is demonstrated by the agreement between the data, which were all taken with one tuning of the dot, and the simulations shown, which were all performed with one set of values for the system parameters.

\(^{2}\)Either or both dots may contain a closed shell beneath the valence electrons; if present, such shells do not appear to play a role in the work we report.
5.2 Coherent oscillations with application of a single pulse

Figure 5.1 demonstrates that oscillations both between states \(|0\rangle\) and \(|2\rangle\) as well as between states \(|2\rangle\) and \(|3\rangle\) can be established and measured by application of the simple pulse
Figure 5.1: (cont.) (h) Top: computed time evolution of the diagonal elements of the density matrix during the pulse for the data point labeled with pink pentagon in (f). The rising edge of the pulse increases the population of states $|2\rangle$ and $|3\rangle$ to 70% and 23% respectively. Bottom: Time evolution of off-diagonal terms in the density matrix for the data point labeled with pink pentagon in (f). Relative phase oscillations between the two states during the $\varepsilon_p$-portion of the pulse are clearly visible. (i) The relative phase $\theta_{23}$ of states $|2\rangle$ and $|3\rangle$, taken at the half point of the falling edge of the pulse, as a function of pulse width and the probability of measuring $(1,2)$ charge occupation as a function of pulse width. The two curves are well correlated with each other, indicating the phase oscillation information during the pulse is mapped to charge probability by the falling edge of the pulse.

sequence shown in Fig. 5.1(b), which has been used in previous work to investigate quantum oscillations between states of a charge qubit \[88, 107, 99, 90, 13\]. The detuning voltage starts at a negative base value, where state $|0\rangle$ is favored energetically (see Fig. 5.1(c)), and then is pulsed to more positive detuning, close to the $|0\rangle$-$|2\rangle$ and $|0\rangle$-$|3\rangle$ anticrossings. After a short time (typically of order 1-10 ns), the pulse ends and the detuning returns to its base value. Fig. 5.1(d) shows the resulting transconductance of the quantum point contact (QPC) indicated on Fig. 1(a), which is sensitive to changes in the time-averaged charge occupation of the dot. If the electron is in one of the $(1,2)$ states, $|2\rangle$ or $|3\rangle$, at the end of the pulse, it remains in that state until it decays inelastically back to a $(2,1)$ state, which takes $\sim 18$ ns \[13\]. Thus, the average charge distribution in the dot reflects the occupation of the dot just after the end of the pulse. Two different types of oscillations are observed and are highlighted in orange and pink in Fig. 5.1(e): the former occur near the anticrossing between $|0\rangle$ and $|2\rangle$ and have a frequency that depends strongly on detuning. The latter arise for $\varepsilon_p$ more positive than the former location and have a frequency that is nearly independent of detuning.

To gain insight into the two different oscillation frequencies shown in Fig. 5.1(d-e), we perform numerical simulations (see Section 5.5 for details) of the dynamics of a system with the energy spectrum shown in Fig. 5.2(a), with low-frequency detuning noise incorporated as in Ref. \[90\]. Fig. 5.1(f) shows the result of the simulation, which is in good agreement with the data. When $\varepsilon_p \approx 0$, the oscillations (highlighted with orange in Fig. 5.1(e)) are between
the states $|0\rangle$ and $|2\rangle$, the ground states of the (2,1) and (1,2) charge occupations. The “sideways-v”, criss-cross pattern of the oscillations in this regime is characteristic of lock-in measurement of charge qubits \cite{107, 99, 90, 13}; it arises because the oscillation frequency depends strongly on $\varepsilon_p$, with a minimum frequency of $2\Delta_1$ at $\varepsilon_p = 0$. At larger values of $\varepsilon_p$, oscillations at a different frequency appear (highlighted by the nearly parallel pink lines near the bottom of Fig. 5.1(e)). These oscillations have a different period ($\sim 100$ ps) that depends only weakly on $\varepsilon_p$; they are well-described by the simulation of Fig. 5.1(f), and their frequency is set by the energy difference between the states $|2\rangle$ and $|3\rangle$. As is clear from the full time evolution of each relevant state, which is plotted in Fig. 5.1(h), at this detuning the rising edge of the pulse transfers the large majority of the weight in the wavefunction into states $|2\rangle$ and $|3\rangle$, leaving very little occupation of $|0\rangle$. An approximate quantum wavefunction during the $\varepsilon_p$-portion of the pulse is thus given by

$$|\psi(t)\rangle \approx e^{i\phi(t)} \left( a|2\rangle + be^{i\delta E_R t/\hbar}|3\rangle \right),$$

(5.1)

with $\phi(t)$ a global phase that does not affect measurable quantities. While the charge sensing measurement does not distinguish between states $|2\rangle$ and $|3\rangle$, the oscillations are visible in this experiment because the two contributions interfere when the pulse ends and the detuning passes back through the two anticrossings shown in Fig. 5.1(c), between $|0\rangle$ and $|2\rangle$ as well as $|0\rangle$ and $|3\rangle$, so that the occupation of the (1,2) charge state after the pulse has ended oscillates with angular frequency $\delta E_R/\hbar$. This relationship between the phase difference between the amplitudes in $|2\rangle$ and $|3\rangle$ and the probability of occupying the (1,2) charge state $P_{(1,2)}$ is illustrated in Fig. 5.1(i). The physical mechanism giving rise to the ability to measure oscillations between two states with the same charge distribution via a time-averaged charge measurement is closely related to Landau-Stückelberg-Zener oscillations \cite{109, 110, 111}, so we will refer to these oscillations as LSZ oscillations.

The figure of merit (the ratio of the coherence time $T^*_2$, extracted from the oscillation decay at times longer than those shown in Fig. 1(d), to the oscillation period) of the LSZ oscillations between $|2\rangle$ and $|3\rangle$ is much larger than that of the charge qubit oscillations between $|0\rangle$ and $|2\rangle$, for two reasons. First, the frequency of the LSZ oscillations is not
limited by the pulse rise time; the oscillation frequency is determined by the energy difference $\delta E_R$ between state $|2\rangle$ and $|3\rangle$; whether a particular pulse rise time results in a state in which $|2\rangle$ and $|3\rangle$ both have substantial occupation is determined by the value of the tunnel couplings $\Delta_1$ and $\Delta_2$. Second, the energy difference $\delta E_R$ depends only weakly on detuning $[11]$, so the LSZ oscillations are less susceptible to the dominant source of decoherence $[112, 90, 99]$, fluctuations in the detuning, than are oscillations between energy levels with different dependences on the detuning, such as the levels $|0\rangle$ and $|2\rangle$ used for a standard charge qubit.

### 5.3 Coherent oscillations with application of complex pulses

We now show that more complex pulse sequences can establish oscillations between different pairs of states in the system, including between states that are both excited states of the system at all values of the detuning accessed during the sequence. Fig. [5.2(b)] shows the measured transconductance of the QPC charge sensor during application of the voltage pulse shown in Fig. [5.2(c)]. Five distinct oscillation patterns can be identified, as shown with the color overlay in Fig. [5.2(c)], and with corresponding colors in Fig. [5.2(a)]. The numerical simulation of Fig. [5.2(d)], which uses the same parameters as the simulation in Fig. [5.1(f)], shows that the oscillation frequencies in the data correspond to energy level differences between specific pairs of quantum states. That mapping is shown in Fig. [5.2(a)], and the validity of our model is demonstrated by the accuracy of Fig. [5.2(d)]. The color overlays in Fig. [5.2(a)] and [5.2(c)] show that, even with a single, relatively simple pulse pattern, quantum superpositions and oscillations can be observed between nearly all possible pairs of states. The relative weight of each of these oscillations, which reflects the relative weight of the wavefunction in each rung in the ladder of energy eigenstates, is determined by the pulse rise time, the pulse detuning, and the tunnel couplings that determine the size of the anticrossings between the three-electron states.
Figure 5.2: A simple multilevel pulse sequence induces quantum oscillations between different pairs of levels at different values of the pulse detuning. (a) Diagram of relevant energy levels of the electrons in the double quantum dot. The oscillations highlighted in the measurements shown in panel (c) correspond to the transitions between levels denoted by the appropriately colored regions in this diagram. The parameters used in the simulation are also listed. (b) Measured transconductance through the charge sensing QPC, which reflects changes in the time-averaged charge state of the double quantum dot as a function of the pulse detuning $\varepsilon_p$ and of the pulse duration $t_p$, with all other parameters held fixed, in the presence of the pulse sequence in (e). Coherent oscillations between different pairs of charge states are reflected in the oscillation of the time-averaged charge occupation in the dot as a function of $t_p$ at different values of the detuning. (c) Same as (b) with the different frequencies highlighted by differently colored lines. The oscillations highlighted here correspond to the transitions between levels denoted by the corresponding colored regions in (a). (d) Results of numerical simulation of the system with parameters listed in (a), in the presence the pulse sequence in (e). The pulse rise time used is 118 ps. (e) A typical pulse trace for this experiment. Inset: The relative position of the pulses with respect to the energy levels for the data point labeled by the green triangle and red diamond in (b). (f) Projection onto the Bloch sphere for the states $|0\rangle$ and $|1\rangle$, with the trajectory of the state vector during the $\varepsilon_p$-portion of the pulse mapped out at $\varepsilon_p = -291 \mu$eV. (g) Computed time evolution of the coupling term between states $|0\rangle$ and $|1\rangle$ for the data point labeled with pink pentagon in (d). Relative phase oscillations between the two states during the $\varepsilon_p$-portion of the pulse are clearly visible.
5.4 Relationship to the quantum dot hybrid qubit

One important reason for manipulating quantum states is to perform quantum information processing. For this application, one needs to create quantum gates, which are unitary transformations. If one defines a qubit as two states \( |\tilde{0}\rangle \) and \( |\tilde{1}\rangle \), then to qualify as a gate, a process that transforms \( |\tilde{0}\rangle \rightarrow a|\tilde{0}\rangle + b|\tilde{1}\rangle \) must transform either \( |\tilde{1}\rangle \rightarrow -b^*|\tilde{0}\rangle + a^*|\tilde{1}\rangle \) or \( |\tilde{1}\rangle \rightarrow b^*|\tilde{0}\rangle - a^*|\tilde{1}\rangle \). Ref. [94] presents pulse sequences that, when applied to a double quantum dot with three electrons, yield any prescribed rotation on the Bloch sphere of a qubit with basis states \( |0\rangle = |\tilde{0}\rangle, |1\rangle = |\tilde{1}\rangle \), where \( |0\rangle \) and \( |1\rangle \) are two of the states we study here. A \( \pi \) rotation that sends \( |0\rangle \rightarrow |1\rangle \) can be implemented by performing successive \( \pi \) rotations at the two successive anticrossings marked \( \Delta_1 \) and \( \Delta_3 \) in Fig. 5.2(a).

Here we can understand the oscillations highlighted green in Fig. 5.2(e) in the language of the hybrid qubit. The first 340 ps pulse in Fig. 5.2(e) rotates significant weight of the wavefunction from \( |0\rangle \) into state \( |2\rangle \), which would be called an auxilliary state in a \( (2,1) \) hybrid qubit. The second, variable section of the pulse pushes the double dot to deep negative detuning, with very different effects on the fraction of the wavefunction in states \( |0\rangle \) and \( |2\rangle \). State \( |0\rangle \) simply slides to lower energy in the detuning plot shown in Fig. 5.2(a). State \( |2\rangle \), in contrast, moves to higher energy, where it anticrosses with \( |1\rangle \). This anticrossing, governed by tunnel coupling \( \Delta_3 \), is large enough that the pulse is largely adiabatic and therefore the majority of the weight in \( |2\rangle \) follows the lower branch to state \( |1\rangle \), whose dependence of energy on detuning is nearly the same as \( |0\rangle \), setting up a superposition whose phase difference is relatively immune to noise in detuning. The second 340 ps pulse in Fig. 5.2(e) reverses this process and drives a second rotation at the \( \Delta_1 \) anticrossing, enabling observation of interference as a function of the evolved phase difference between states \( |0\rangle \) and \( |1\rangle \).

The figure of merit for the resulting oscillations is over 100, an extremely high value for the present state of semiconductor qubits. Thus, the oscillations shown in green reflect a controlled phase evolution between states \( |0\rangle \) and \( |1\rangle \), a \( \hat{z} \)-rotation for the hybrid qubit, demonstrating a key ingredient in constructing a pulse-gated quantum dot hybrid qubit [94].
5.5 Simulation Method

Numerical simulations of the experiment were performed based on the energy level diagram in Fig. 5.2(a) using the pulse rise time of 118 ps. We model the dynamical evolution of the density matrix $\rho$ of the system as a function of detuning $\varepsilon$ and pulse duration $t_p$ using a master equation [95]:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho]$$

(5.2)

The Hamiltonian, written in the basis of position eigenstates, is

$$H = \begin{pmatrix}
\frac{\varepsilon}{2} & 0 & \Delta_1 & -\Delta_2 \\
0 & \frac{\varepsilon}{2} + \delta E_L & -\Delta_3 & \Delta_4 \\
\Delta_1 & -\Delta_3 & -\frac{\varepsilon}{2} & 0 \\
-\Delta_2 & \Delta_4 & 0 & -\varepsilon/2 + \delta E_R
\end{pmatrix}.$$  

(5.3)

The (1,2) charge occupation probability is extracted at the end of the pulse and is averaged over 2 ns in the measurement stage of the pulse. Using this number as an initial value, $P_{(1,2)}$ is then allowed to relax exponentially to the ground state (2,1) occupation with a relaxation time $T_1$, during the measurement phase. Finally, the simulated charge occupation is determined by averaging the charge state for the entire 33 ns pulse period. Low-frequency fluctuations in the detuning $\varepsilon$ are incorporated following Ref. [90], by performing a convolution of the results at each $\varepsilon$ with a Gaussian in $\varepsilon$ of width $\sigma_\varepsilon = 5 \mu$eV. The best fit to the data is found with a charge $T_1 = 18$ ns.

5.6 Determination of parameters

In the simulation, there are six important parameters, they are tunnel couplings $\Delta_1$, $\Delta_2$, $\Delta_3$, $\Delta_4$ and excited states energies $\delta E_L$ and $\delta E_R$. Among them, $\delta E_R$ is determined by the Larmor oscillation frequency at large positive detunings (fig 5.1(e) pink lines) and $\delta E_L$ is determined by the oscillation frequency at large negative detunings (fig 5.2(c) green lines). In this experiment we use $\delta E_R = 9.2$ GHz and $\delta E_L = 52.7$ GHz. In addition, $\Delta_1$ can be
Figure 5.3: Simulation of oscillations with application of a single pulse using different values of $\Delta_2$. (a) Experimental data, same as Fig 5.1(d). (b) Simulated data, same as Fig 5.1(f), using the parameters in Fig 5.2(a), with $\Delta_2 = 3.5$ GHz. Good agreement with (a) is found. (c) Simulated data, using the same parameters as (b) except $\Delta_2 = 1$ GHz. The visibility of LSZ oscillations are far less strong than the data in (a). (c) Simulated data, using the same parameters as (b) except $\Delta_2 = 6$ GHz. The anti-crossing is shifted toward positive detuning. The frequency of the oscillations is also deviated from the data in (a).

determined directly from the Larmor oscillation frequency at $\varepsilon = 0$(Fig 5.1(e) orange side-V shapes), which is 2.62 GHz.

The determination of $\Delta_2$ is not trivial. The value of $\Delta_2$ has no significant influence in the oscillation frequency at either 0 or large positive detuning, rather, it makes a difference to the appearance or visibility of the LSZ oscillations, as shown in Fig 5.3. However, simulation suggests that when $\Delta_2$ is within certain range($3 \sim 5$ GHz), the simulated data is about equally similar as the experiment data. On the other hand, the value of $\Delta_2$ also has an impact on the appearance of Fig 5.2(b) at small negative detunings. The value
Figure 5.4: Larmor oscillations at excited anti-crossings. (a) Larmor oscillation at the anti-crossing between state $|1\rangle$ and $|3\rangle$, with the presence of pulse in (c). (b) Relative position of the pulse and energy diagram. (c) A typical pulse trace for this experiment. $V_{p1}$ is kept at 600 mV and $t_{p1}$ is kept at 360 ps.

The determination of $\Delta_2$ determines the oscillation appearance within $-20 \sim -80$ $\mu$eV range in fig 5.2(b). Simulation shows that for a large $\Delta_2$ (e.g. 5 GHz), the oscillation frequency at around $\varepsilon = -50$ $\mu$eV is too fast. For this reason, we use the value 3.5 GHz for $\Delta_2$.

Figure 5.4 shows how we determine $\Delta_4$. Start from the state $|0\rangle$, the pulse first takes the system to a little pass $\varepsilon = 0$ and stay there for a while, during which the population of state $|0\rangle$ decreases and population of $|2\rangle$ and $|3\rangle$ increase. The pulse then takes the system to a negative detuning around the anti-crossing between $|1\rangle$ and $|3\rangle$, this allows the population of state $|3\rangle$ converts to state $|1\rangle$. Finally, the system is pulsed back to the measurement position where different charge populations can be projected. Varying the time spent around the anti-crossing between $|1\rangle$ and $|3\rangle$ results in oscillations between state $|1\rangle$ and $|3\rangle$, and the oscillation frequency determines the value of $\Delta_4$. In this experiment, we use $\Delta_4 = 1.65$ GHZ.

The determination of $\Delta_3$ is again nontrivial. The reason we observe the oscillations in Fig 5.2(c) with green color label is partly because $\Delta_3$ is big, so that the first falling edge of the pulse is able to send the population in state $|2\rangle$ to state $|1\rangle$, allowing the interference...
Figure 5.5: Simulation of oscillations with application of complex pulses using different value of $\Delta_3$. (a) Experimental data, same as Fig 5.2(b). (b) Simulated data, same as Fig 5.2(d), using the parameters in Fig 5.2(a), with $\Delta_3 = 4.6$ GHz. Good agreement with (a) is found. (c) Simulated data, using the same parameters as (b) except $\Delta_3 = 2.2$ GHz. The oscillations for $\varepsilon_p < -50 \mu$eV have substantial different appearance than those in (a). (d) Simulated data, using the same parameters as (b) except $\Delta_3 = 8$ GHz. Again, the oscillations for $\varepsilon_p < -50 \mu$eV deviate a lot from those in (a).
between state $|0\rangle$ and $|1\rangle$. After running the simulation for different values of $\Delta_3$, we find a good fit to the experiment data at $\Delta_3 = 4.6 \text{ GHz}$.

### 5.7 Summary

We have demonstrated that high-speed voltage pulses can be used to control coherent quantum oscillations between different pairs of states in a semiconducting double quantum dot with three electrons. By implementing appropriate combinations of voltage pulses, oscillations between different pairs of levels as well as sequential operations can be achieved. Transitions between some pairs of levels can be induced that have over a hundred oscillations within a coherence time. All of the observed rotations and oscillations have frequencies in excess of 5 GHz. These results provide strong evidence that coherent, fast oscillations can be initiated and controlled between multiple levels of three electrons in a double quantum dot.
Chapter 6

Conclusion

In this thesis, we have explored both spin and charge properties of Si quantum dot qubits. We proposed a hybrid qubit scheme and performed preliminary experiments on it.

The first chapter provided introduction and basic background of quantum computation and semiconductor quantum dots.

In Chapter 2, we used a magneto-spectroscopy method to measure the excited-state spectrum of a Si/SiGe quantum dot and extracted the singlet-triplet energy splitting. We found it to be tunable by lateral displacement of the dot. We presented calculations showing the data are consistent with a spectrum in which the first excited state of the dot is a valley-orbit state.

Chapter 3 utilized the singlet-triplet energy splitting in the left quantum dot measured in Chapter 2 together with one electron in the right quantum dot, to propose a fast hybrid Si double dot qubit. The hybrid qubit architecture has an attractive combination of speed and fabrication simplicity. Moreover, the system has potentially long decoherence times. We measured a $T_\tau$ to $S$ relaxation time in a single silicon dot to be $> 100$ ms, and we demonstrated rate dependent read-out of the singlet and triplet states of two electrons in a silicon dot. We estimated dephasing times theoretically to be on the order of microseconds, long enough to achieve high fidelity quantum operations.

As a preparation for implementing the hybrid qubit, in Chapter 4 we demonstrated
coherent oscillations and echo measurements in a Si charge qubit. The measured inhomogeneous dephasing time $T_2^*$ ranged from 127 ps to 2.1 ns; it depended substantially on how the energy difference of the the two qubit states varied with external voltages, consistent with a decoherence process that is dominated by detuning noise. In the regime with the shortest $T_2^*$, applying a charge-echo pulse sequence increased the measured inhomogeneous decoherence time from 127 to 760 ps, demonstrating that low-frequency noise processes are an important dephasing mechanism.

Finally in Chapter 5, we demonstrated coherent manipulation of the hybrid qubit. We demonstrated that tailored pulse sequences can be used to induce coherent rotations between 3-electron quantum states. Certain pulse sequences yielded coherent oscillations with a very high figure of merit (the ratio of coherence time to rotation time) of $>100$. These oscillations can be interpreted as z-rotations of a hybrid qubit. The presence of the third electron enables very fast rotations to all possible states, in contrast to the case when only two electrons are used, in which some rotations are slow. This demonstrates a promising future for the Si hybrid qubit.
Bibliography


